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Prospect of employing conductors at low temperature in power cables and in power transformers

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Synopsis

An estimate is made in economic terms of power that could be saved if the conductor in a 760 MVA, 275 kV a.c. cable were alternatively:

niobium at 4°K
aluminium at 20°K
beryllium at 77°K

The first of these metals is a soft superconductor, and the others, if sufficiently pure, are assumed to exhibit resistivities of 3×10^{-9} and $2 \times 10^{-8}\Omega$cm, respectively. The analysis takes into account the effects of eddy currents at these resistivities, and considers the spacing of refrigerating units along the cable and the distribution of refrigerants in order to minimise that part of the cost that stems from refrigeration.

The study turns next to a 570 MVA generator transformer with aluminium at 20°K and resistivity $3 \times 10^{-9}\Omega$cm, or, alternatively, a hypothetical metal at 77°K and resistivity $2 \times 10^{-8}\Omega$cm, as the winding conductor. The paper finds that these metals would both have to be used in the form of foil, and derives expressions for three components of foil loss. It then compares the possible saving in conductor loss with the drive power used by the refrigerators, and with the cost of refrigeration plant. It finds that neither in power transmission by cable, nor in power-transformer windings, is there any promise of a net saving over conventional costs that would warrant the constructional and operational complexities entailed in deep refrigeration.

List of symbols

a = effective radial spacing of a coil pair, cm
b = radial build of a coil, cm
B_0 = flux density at a radial position z, Gs
$\hat{B}$ = peak leakage flux density in the intercoil path, Gs
B' = radial peak flux density near the coil ends, Gs
C, C_0 = cost of refrigerating plant capable of the duty H or H_0, £
$\overline{C}_m$ = cost of refrigerating plant per km of cable when optimised, £/km
d = outer diameter of a cylinder receiving radiant heat, cm
d' = distance between the end of a coil and an arbitrary yoke ceiling, cm
D = coefficient concerned in eddy loss, cm^{-2}
E = term related to conduction loss, cm^{-2}
F = term related to eddy loss, cm^{-2}
f = factor defining fluid loss
h = ratio between eddy and conduction losses in a coil
h_e = electrical loss in the cable conductor, W/km
h_f = fluid loss, W/km
h_r = radiant inleak of heat, W/km
h_t = sum of electrical and inleak power to a cable that is to be removed by a refrigerant, W/km
H = heat to be pumped from a stated length of cable core, W
H_0 = heat capacity of plant costing C_0, W
I = r.m.s. current, A
I_0 = peak current, A
I_b = edge current in a foil-wound coil, A
J_0 = peak current density corresponding to I_0, A/cm^2
j, j_0, j_e = instantaneous current densities, A/cm^2
k = fraction of a coil space occupied by wire
l = axial length of a winding, cm
l' = length of cable between refrigerating units, km
l'_m = optimum value for l', km
m = number of single cables that are served by each refrigerating unit
M = gross volume of a coil, cm^3
n = power of H that is proportional to plant cost
N, N_1, N_2 = turns in a coil
p = coefficient which is equal to 1 if fluid flows only one way and is equal to 2 if fluid returns in the same cable
r = inner radius of the core, cm
R = outer radius of the core, cm
s = radius of wire strand, cm
t, t_1, t_2 = mean length of turns, cm
T = temperature of a radiating surface, degK
u = section of a refrigerant channel, cm^2
v = fluid velocity, cm/s

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v_m = optimum fluid velocity, cm/s
P = loss in general, W
P_b = loss at the borders (edges) of foils, W
P_c = combined conduction, eddy, and plain-edge loss in a coil, W
P_{cr} = combined conduction, eddy, and curled-edge loss in a coil, W
P_r = loss at the curled edges of a coil, W
P_t = loss in one turn, W
α = fraction of r that is the radial thickness of a hollow core
β = average error of radial position in a transposed conductor system, cm
γ = average radial excursion of cable strands in fully transposing a layer, cm
δ, δ_m = foil thickness, cm
Δ = edge-current (axial) penetration, cm
θ = an allowable excursion in refrigerant temperature, deg K
μ = permeability
ρ = resistivity, Ωcm
σ = specific heat, J/cm^3 deg C
ω = 2π × frequency

1 Introduction

The resistance of pure metals falls with temperature, and in favourable cases can become 900 times less than normal at a temperature that is still considerably above those required by superconductors. A review has recently been made in France[1] of the application of pure metals such as aluminium and beryllium, with their resistivity depressed by refrigeration, to the windings of transformers and alternators. The French work included the construction of an experimental transformer with a potential rating of 500kVA, in which the windings were of aluminium foil maintained at 20°K by liquid hydrogen. Tests with direct current gave an effective resistivity of $3 \times 10^{-9}\,\Omega$cm, and 50c/s current was circulated under short-circuit conditions.

In the light of the French work and of rising interest in this country in the subject of metals at low temperature, the intention of the study is to examine in outline the technical and economic merit of using pure aluminium at 20°K, or metal with a resistivity of $2 \times 10^{-8}\,\Omega$cm at 77°K, as the conductor, either in power transformers or cables, and of using niobium at 4°K in cables. The course of the inquiry will cover the arrangement of conductors for minimum loss, and the danger of transient-fault overloads in some designs. It will also review the thermal load, the power requirements, and likely cost of refrigeration plant.

2 An essential condition of merit

Any advantage that could arise from incorporating metals with high conductivity must be seen as a saving in conduction loss, which, when expressed as a capital sum, outweighs the collective costs of refrigeration plant, the cost of power to drive this plant, and any increase in the cost of manufacture of the cable or transformer. Allowance must be made for differences in the cost of materials that may arise through the change from copper.

The capitalised value of the losses in any piece of electrical equipment must take into account the cost of generating and bringing energy to that equipment, the load factor in current-dependent loss, and interest charges on capital.[2] Some recent assessments by the CEGB have been £145/kW for I^2R loss in a generator transformer, £220/kW for transformer-core loss, and £60/kW for I^2R loss in a substation transformer. For simplicity of illustration here, it will be assumed that all conductor loss corresponds to £100/kW and continuous losses to £220/kW.

It is apparent that refrigerated cables and refrigerated transformer windings would be convenient partners, but it is unlikely that a decision to refrigerate one of them would be materially influenced by the temperature of the other. This is because the difficulty of bringing current from a refrigerated conductor to one at room temperature is small when compared with the general problem of refrigeration, whether of a cable or of a transformer.

3 Cables with refrigerated conductors

In a refrigerated cable, account must be taken of heat entering the core from three sources, which are:

- electrical losses that are collectively generated in the core
- heat radiating into the core from surroundings at normal temperature
- energy dissipated by viscous losses in the flowing refrigerant.

Dielectric losses are not included, because, to avoid this extra refrigeration burden, it would be reasonable to place the necessary thermal insulation and its refrigerant system next to the core and inside the dielectric.

Consideration will now be given to thermal insulation and inleak rates, fluid losses, electrical losses in the core, and cost of refrigeration plant, as a collective basis upon which to make optimum assessments. For convenience of comparison, the findings will then be referred to a 3-phase circuit rated 760MVA at 275kV 1600A, that now uses three separate cables with a copper conductor of 3in^2 section in each.

3.1 Thermal insulation and inleak rates

It is prudent to seek the highest reasonable thermal insulation, and a practicable solution to the problem of supporting a cable core without providing a large path for heat conduction would have to be worked out in some detail. However, an answer to this problem will be assumed to be provided by small-gauge struts of high tensile strength and low thermal conductivity. Estimates based on radial spokes of glass fibre placed at a rate of 12 to the metre gave rise to an inleak by conduction that is small, even compared with coolant-screened radiation; so heat leaking through conduction will be ignored. The thermal insulation will be assumed to be provided in steps, each consisting of a highly evacuated annular space, concentric with the core and subdivided into three by two intervening and isolated radiation shields. The six reflecting surfaces involved in one step of thermal insulation of this sort would be of highly reflecting aluminium with the low emissivity coefficient of 0·023; heat is then radiated from the outer reference surface of the thermal insulation at temperature T (Fig. 1) to its inner insulation boundary of diameter d, approximately in accordance with

$$h_r = 1{\cdot}44 \times 10^{-8} d T^4 \qquad (1)$$

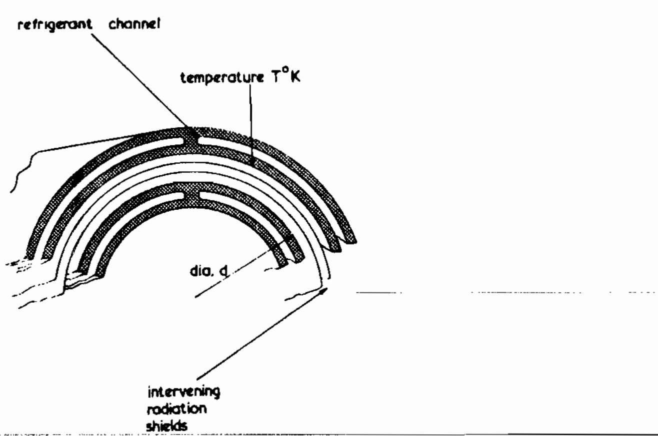

Fig. 1
Arrangement of annular cooling jackets with the evacuated intervening space occupied by two radiation shields

3.2 Multiple-refrigerant cooling

As shown in Table 1, the power consumed in transporting heat from any low temperature to a normal temperature increases acutely as the lower temperature falls.

Table 1

THE EFFICIENCY OF REFRIGERATORS FOR WHICH THE INPUT IS 200 KW OR ABOVE. THE EFFICIENCIES ARE OPTIMISTICALLY ASSUMED TO HOLD ALSO FOR SMALLER UNITS

Liquid refrigerant	Temperature	Drive power per watt transported		
		Carnot ideal	Large systems	
			Estimated future practice	Present practice
	deg K	W	W	W
Helium	4	65	300	620
Hydrogen	20	11·5	39	78
Nitrogen	77	2·5	7·4	14

In the severe example of a helium-cooled (superconductor) core, it is clear from eqn. 1 that the heat radiated onto it from surroundings at the temperature of liquid hydrogen is about 15^4, or 50000 times less than if the same surroundings were raised to room temperature. Moreover, Table 1 shows that, to save drive power, it is more important to restrict incoming heat at the lower temperatures than at intermediate ones. These two considerations point to the provision of multiple-refrigerant cooling, and Fig. 2 shows the use of three refrigerants, namely liquid helium, liquid hydrogen and liquid nitrogen, used with a niobium-foil conductor.

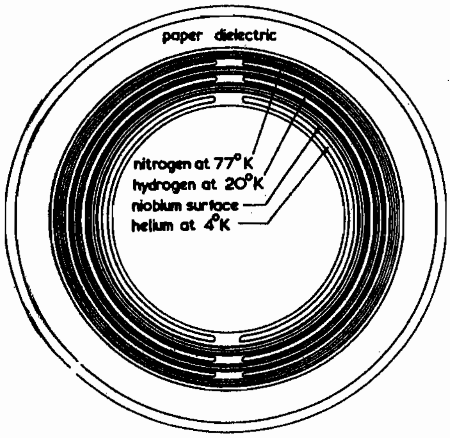

Fig. 2
Superconducting thin-walled niobium core cooled internally by liquid He and protected externally by liquid H_2 and liquid N_2

3.3 Conditions for the optimum circulation of refrigerant

The flow of any one refrigerant would take place in a concentric annular duct within a cable, and different fluids would occupy the radial positions appropriate to their temperatures. The return of any one fluid to its refrigerating station must therefore take place either in a second duct at the proper radial position in the same cable, or in a second cable. The twin-duct arrangement is indicated in Fig. 3 and the use of paired cables in Fig. 4.

It will be evident that, if the total duct section in a cable is fixed, conditions remain unchanged as far as refrigerator spacing, fluid losses, and minimum cost of plant are concerned. A choice between the twin-duct and cable-pair systems can therefore be left to considerations that are not relevant in this review, though it is to be noted that twin ducts enable the bridging of voltage by fluid to be restricted to one phase, and are not limited, as are whole ducts, to an even number of cables. Fig. 5 is drawn to describe either system, according to whether p is 1 for the whole duct, or 2 for the twin duct, and the effect of varying the route distance will now be discussed.

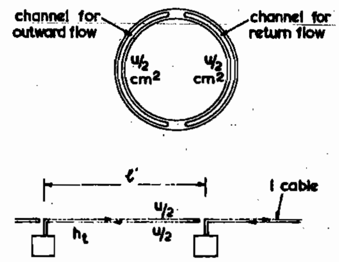

Fig. 3
Coolant circulation in a twin-duct jacket

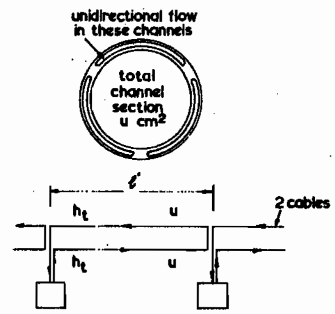

Fig. 4
Coolant circulation using two cables

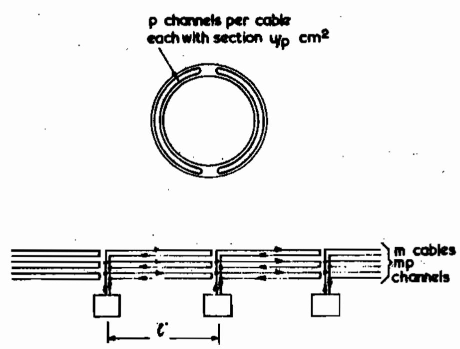

Fig. 5
Generalised circuit for the refrigerant

The cost of any one refrigerant unit is related to its specified performance by an expression of the form

$$C = C_0\left(\frac{H}{H_0}\right)^n \qquad (2)$$

where the index n is less than unity. This shows that considerations of the cost of plant would favour long intervals between pumping stations.

On the other hand, because the amount of heat that can be transported by a given volume of liquid is limited, fluid velocity, and consequently viscous losses per kilometre, must rise with distance; so it follows there is some route interval between stations at which the cost of pumping plant per km of cable becomes a minimum.

Turning to Fig. 5, we see that the rate at which heat is to be removed by the refrigerant from a length of m cables laid together is

$$H = ml'(h_t + h_f) \qquad (3)$$

where $h_t = h_e + h_r$ . . . . (4)

The operating velocity at which liquid passes through the cable duct must be so arranged that the rate of heat transfer does not cause an excessive rise in fluid temperature, and a change to the gas phase. The allowable temperature rise, fluid velocity and channel section are then related by

$$H = mu\sigma\theta v/2 \quad . . . . (5)$$

At velocities that are of interest, the liquid flow is turbulent and the energy dissipated as heat in this way may be written

$$h_f = fuv^3 \quad . . . . (6)$$

where f is a factor only slightly dependent on v. For example, in the case of liquid hydrogen, a tenfold increase in velocity from 10cm/s to 100cm/s causes only a twofold drop in f. Accordingly, f will be regarded as a constant to be checked by successive approximation.

These six relationships apply separately to each refrigerant, and from them it can be deduced that the fluid velocity and spacing between refrigerator stations that lead to a minimum refrigerator cost per km of cable are, respectively, expressed by

$$v_m = (h_t/uf)\{(1-n)/(2+n)\}^{1/3} \quad . . . . . (7)$$

and $$l_m = (2+n)^{2/3}(1-n)^{1/3}u^{2/3}\sigma\theta/6h_t^{2/3}f^{1/3} \quad . . (8)$$

and the minimum cost per km of single cable is itself

$$\bar{C}_m = \frac{3C_0}{H_0^n}\left(\frac{h_t}{2+n}\right)^{\frac{2+n}{3}}\left(\frac{f}{1-n}\right)^{\frac{1-n}{3}}\left(\frac{2}{mu^{2/3}\sigma\theta}\right)^{1-n} \quad (9)$$

It also follows that

$$h_f = h_t(1-n)/(2+n) \quad . . . . (10)$$

With nitrogen, $n = 0{\cdot}76$, and these expressions then become

$$v_m = 0{\cdot}444(h_t/fu)^{1/3} \quad . . . . (11)$$

$$l'_m = 0{\cdot}204\sigma\theta u^{2/3}/f^{1/3}h_t^{2/3} \quad . . . . (12)$$

$$\bar{C}_m = 1{\cdot}56C_0h_t^{0\cdot92}f^{0\cdot08}/(m\sigma\theta)^{0\cdot24}H_0^{0\cdot76}u^{0\cdot16} \quad . (13)$$

and $$h_f = 0{\cdot}085h_t \quad . . . . (14)$$

3.4 A superconducting cable

It is now possible to assess the economic merit of transmitting power by cables where there are no losses of an electrical nature. As the magnetic field in cables is generally low, a soft superconductor can be considered, and the best conductor in this respect is annealed niobium with a critical field of 1550Gs.[3] A.C. losses in niobium are finite,[4] but in the design that is considered they would only amount to 1mW/km at 1600Ar.m.s., and this can be neglected.

If the worst fault is modestly assumed to be 7600MVA, fully offset current would reach 40kA peak, and the field of 1550Gs requires that the niobium conductor should take the form of a thin-walled cylinder, 10·4cm in diameter. Fig. 2 shows such a core, cooled internally by liquid helium and externally be a stage of hydrogen and, beyond that, by nitrogen, with three pairs of intervening radiation shields. It is likely that making and laying this structure would be unwarrantably expensive, but it is nevertheless instructive to apply the system considerations expressed in the last few equations. To do this, it will be assumed that six cables, the members of two circuits which follow the same route, are served by common refrigerating units. The resulting values are listed in Table 2 and lead to a minimum plant cost of £4313/km of single cable.

This cost requires refrigeration units to be placed at their optimum intervals, and, because in the case of helium the optimum value of 69·7km would be too great to be operationally acceptable, the combined plant cost must somewhat exceed the minimum.

The elimination of liquid hydrogen is a desirable step on grounds of both simplicity and safety, but this necessarily increases the heat load in helium, and, as shown in Table 3, it leads to an increase of 47% in the combined plant cost to £6370/km of cable.

Table 2

OPTIMUM COSTS PER KM AND REFRIGERATOR-UNIT SPACING CALCULATED FOR A NIOBIUM CABLE CONDUCTOR THAT IS COOLED WITH STAGES OF HE, H_2 AND N_2

Parameter	Helium	Hydrogen	Nitrogen	Units
u	2·5	3	3·5	cm^2
f	$1{\cdot}9 \times 10^{-4}$	$1{\cdot}35 \times 10^{-4}$	$1{\cdot}0 \times 10^{-3}$	
d	10·4	11·6	12·2	cm
T (Fig. 1)	20	77	300	degK
σ	0·355	0·424	1·76	J/cm^3 per degC
θ	2	3	8	degC
C_0	50000	300000	16000	£
H_0	100	60000	10000	W
n	0·4	0·4	0·76	
h_t	0·024	5·85	1490	W/km
v_m	2·32	14·4	33·2	cm/s
l'_m	69·7	4·00	0·540	km
$\bar{C}_m$	53·0	1200	3060	£/km

Table 3

OPTIMUM COSTS AND REFRIGERATOR SPACING FOR A NIOBIUM CABLE COOLED WITH STAGES OF HE AND N_2

Parameter	Helium	Nitrogen	Units
u	2·5	3	cm^2
f	$1{\cdot}10 \times 10^{-4}$	$1{\cdot}10 \times 10^{-3}$	
d	10·4	11·6	cm
T	77	300	degK
h_t	5·23	1355	W/km
v_m	16·8	33·0	cm/s
l_m	2·34	0·524	km
$\bar{C}_m$	3470	2900	£/km

It is to be noted that, if the conductor diameter were smaller, the cable rating would be proportionately reduced, but that the cost per km, being dependent on $h_t^{(2+n)/3}$, would fall less than the rating. Eqn. 13 shows that the channel section available for liquid plays a relatively small part in controlling the specific cost of plant.

3.5 Aluminium and beryllium conductors at low temperature

In this inquiry it will be assumed that aluminium with 10 parts in 10^6 impurity presents a resistivity[5] at 20°K of 3×10^{-9}cm, and that beryllium with less than 600 parts in 10^6 (mostly BeO) has a resistivity at 77°K, and in magnetic fields up to 500Gs, of $2 \times 10^{-8}\Omega$cm.*

As resistivity falls, losses due to conduction fall with it, but those that arise from the cable's own magnetic field become relatively more prominent. These are of two kinds: one, which can be called eddy-current loss, is characterised by a nonuniform distribution of current over any single member of the cable core; the other, which may be looked upon as a transposition loss, results from unequal sharing of the current between different core members that are nominally in parallel.

In the niobium cable just considered, conduction was restricted to the outer surface of one tube. But a single thickness of foil, in which full current penetration is possible, would be inadequate, and—electromagnetically speaking—there are two choices: these are to make the conductor either in the form of multiple coaxial foil cylinders or from multiple

* BURNIER, P.: Private communication

wire strands. A thin foil that lies parallel to a magnetic field has the advantage of presenting a small section to flux, and to this extent a core with several insulated layers of foil would be reasonably free from eddy current. However, the practical difficulty of causing any one foil to occupy, in turn, the appropriately different radial depths in the core space would be very great, and consideration will therefore be given to the employment of wire strands.

This arrangement lends itself more readily to transposition, i.e. to the positioning of every member of the core so that over the cable's length all strands perceive the same flux linkage. A direct approach is to ensure that, in a stranded core, individual layers retain their identity and that every layer has an equal occupation, in sequence, of all the radial positions.

Loss from faulty transposition can be shown to be of the order β^2/γ^2 of the conduction—or nominal ohmic—loss, where β is the effective error of displacement in a radial direction from the ideal position, and γ is the average radial excursion needed for proper transposition. While the term β^2/γ^2 cannot be zero in a core of circular section that is fully populated with strands, this shortcoming is not by itself likely to cause a serious addition to the cable's loss. On the other hand, the important eddy-current losses tend to be smaller when a stranded core is hollow, and as the analysis can be simplified by considering hollow cores of annular section in which ideal transposition is possible, this will now be done.

Fig. 6 indicates a hollow core where there are the same number of wires in each layer, and it is intended that every layer shall spend the same percentage of cable length at

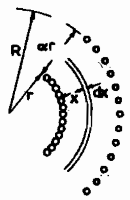

Fig. 6

Section of hollow stranded core

To allow full transposition of strands between r and R, the same number of strands occupy each layer

each radial position. Insulation is assumed, both of layers and of transpositions, but for simplicity the thickness of this insulation is ignored, as is the difficulty of transposing layers.

It is shown in the Appendix that loss/cm in such a cable is given by

$$P = I_0^2 \rho (E + F)/\pi^2 \quad . . . . (15)$$

where the conduction term is E and the eddy component F, and these are, respectively,

$$E = (1 + \alpha)^2/R^2\alpha \quad . . . . (16)$$

and

$$F = \frac{D}{\alpha}\left\{\frac{2 + \alpha}{1 + \alpha} - \frac{2}{\alpha}\log_e(1 + \alpha)\right\} \quad . . . . (17)$$

where $D^{1/2} = 10^{-9}\pi^2\mu\omega s/2\rho \quad . . . . (18)$

The parameter α is defined in Fig. 6, and curves of E and F/D, together with curves of $E + F$, are drawn in Fig. 7. An outer core diameter of 6cm is chosen as a limit set by considerations of handling and laying the cable; so $R = 3$cm.

The curves in Fig. 7 show that conduction loss is minimised when $\alpha = 1$ and that the eddy component F reaches a maximum when α is about $1 \cdot 5$. More significantly, they show the importance, where α is small, of any steps that reduce F, and, as eqns. 17 and 18 indicate, such steps are limited to making the wire section (radius s) small.

It is clear that s must be smaller than the characteristic penetration depth for the resistivity concerned ($700\rho^{1/2}$cm at 50c/s), and to reduce eddy-current losses to the extent indicated by the $(E + F)$ curves in Fig. 7, values have been chosen for s that are given in Table 4. This Table includes, for comparison, a copper conductor at 340° K (67° C) designed on the same basis as the refrigerated cores; i.e. with a hollow centre having $\alpha = 1$ and with the same number of strands in each layer. Its performance must be expected, on this account, to be rather worse than that of a conventionally designed cable. At a point representing the resistivity of hot copper ($\rho = 2 \times 10^{-6}\Omega$cm), E and $(E + F)$ form virtually the same curve, with a minimum value when $\alpha = 1$. With beryllium strands of 12mils diameter ($s = 0 \cdot 015$cm) at 77° K, the optimum value for α is still unity, but eddy-current losses have now risen to 35% of the conduction loss. Aluminium wires at 20° K that are only 4mils in diameter show a

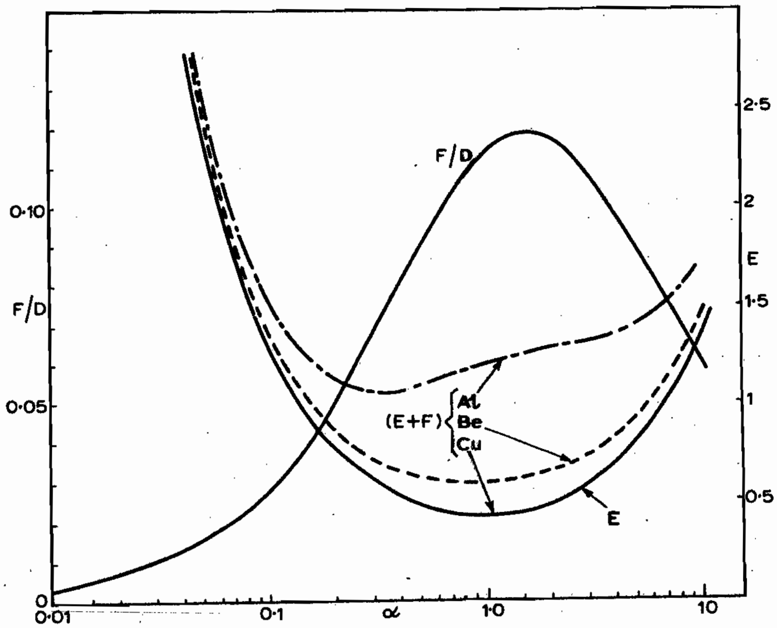

Fig. 7

The coefficients of conductor loss E, and eddy-current loss F

α is defined in Fig. 6
D is independent of α

Table 4

A SUMMARY OF LOSSES AND CONDUCTOR ARRANGEMENT IN CABLES MADE WITH FOUR DIFFERENT CORE RESISTIVITIES

Metal	Temperature	Assumed resistivity	Penetration	Outer-core diameter	Strand radius, s	Strand number	Conductor loss, h_e		
							Ohmic conduction	Eddy-current loss	Total loss
	deg K	Ωcm	cm	cm	cm		W/km	W/km	W/km
Cu	340	2×10^{-6}	1·0	6·0	0·15	160	46 200	300	46 500
Be	77	2×10^{-8}	0·1	6·0	0·015	16 000	460	160	620
Al	20	3×10^{-9}	0·038	6·0	0·005	100 000	470	121	168
Nb	4	0	—	10·4	—	—	—	—	0

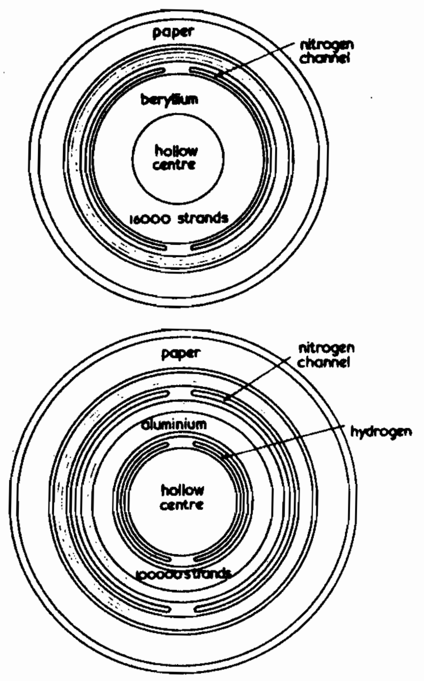

Fig. 8
Two optimum stranded-core designs
Outer conductor diameter of each equals 6cm, and their electrical loss at 1600 A is given in Table 4

minimum combined loss when $\alpha = 0 \cdot 30$ and when the eddy-current component is 55% of the conduction loss.

In effect, the virtue of core hollowness increases with conductivity; for the more hollow a core the less flux density do its members experience at any given depth below the surface. In the limit of extreme conductivity, the optimum value for α becomes vanishingly small, and the result is a thin shell. With aluminium at $3 \times 10^{-9}\,\Omega$cm, the eddy contribution at 50c/s demands a relatively hollow core, as shown in Fig. 8, where it is compared with a beryllium design; it is this feature which tends to offset the nominal advantage of the high conductivity of aluminium.

The four core designs are summarised in Table 4.

3.6 Economic comparisons

The conductor loss, as defined in eqn. 15 and recorded in Table 4, enables an assessment both of minimum plant costs and of optimum spacings for the various refrigerants to be made. Table 1 leads to an estimate, in turn, of refrigerator-unit drive power (based on an expected future achievement in the performance of larger refrigerators, but optimistically applied here to smaller ratings), and from that to a capitalised charge (at £220/kW) for refrigerator power.

In the analysis, a double-circuit 275/kV link is considered with each circuit rated at 760 MVA. Thus in eqn. 13, m equals 6, but power loss, pumping power and costs remain expressed in terms of a single cable. These values are listed in Table 5.

Before accepting the figure of £7017/km for niobium with three refrigerants, it is to be noted that the helium load is so

Table 5

MINIMUM PLANT AND DRIVE-POWER COSTS (ALLOWING FOR CABLE LOAD FACTOR WHERE APPROPRIATE), CORRESPONDING TO NIOBIUM, ALUMINIUM AND BERYLLIUM CORES

Core metal	Refrigerant		Losses				Pump drive power			Costs			
		l'_m	h_r	h_e	$h_r + h_e$	h_t	factor (future)	no load	load part	no load at £220/kW	load part at £100/kW	$\bar{C}_m$	Total cost
		km	W/km	W/km	W/km	W/km		kW/km	kW/km	£/km	£/km	£/km	£/km
Nb	He	69·7	0·024	—	0·024	0·030	300	0·009	—	2	—	53	55
	H_2	4·0	5·85	—	5·85	7·31	39	0·285	—	62	—	1200	1262
	N_2	0·540	1490	—	1490	1630	7·4	12·00	—	2640	—	3060	5700
													7017
Nb	He	2·34	5·23	—	5·23	6·54	300	1·97	—	433	—	3470	3903
	N_2	0·524	1355	—	1355	1470	7·4	10·9	—	2400	—	2900	5300
													9203
Al	H_2	0·402	3·03	168	171	213	39	1·80	6·55	395	655	16 800	17 850
	N_2	0·585	840	—	840	917	7·4	6·80	—	1500	—	1910	3410
													21 260
Be	N_2	0·658	793	620	1413	1540	7·4	6·80	4·6	1500	460	3210	5170

low in this case that the minimum-cost spacing for helium stations becomes 69·7km. As this distance is not practicable, it would be prudent to allow for more frequent helium stations, and so lead to a plant and drive-power cost that is closer to the two-refrigerant cost of £9200/km.

On the credit side, I^2R loss in conventional 3in², 1600A, 275kV (760MVA) cable is about 32kW per km of single cable, and this will be assumed capitalised at £100/kW, a value which is intermediate between £145/kW for generator-transformer I^2R loss and £60/kW for a substation transformer. This leads to £3200/km of cable with which to meet the difference in costs between a refrigerated system and a conventional one.

3.7 Cost of conductor material

The approximate costs of metals when drawn or rolled in quantities sufficient for 1km of the core are listed in Table 6, and comparative costs are given in Table 7.

Table 6

APPROXIMATE COSTS OF ALUMINIUM, BERYLLIUM AND NIOBIUM CONDUCTORS COMPARED WITH COPPER (AT £500/TON IN INGOT FORM) FOR THE SAME DUTY

Metal	State	Approximate cost	
		per lb	per km of core
	mil	£	£
Al	4 (wire)	1·5	3000
Be	12 (wire)	200	800000
Nb	2 (foil)	100	3000
Cu	120 (wire)	0·27	10000

Table 7

A COMPARISON OF COSTS, EXCLUDING CONSTRUCTION AND LAYING, BUT INCLUDING THOSE OF LOSSES, REFRIGERATION PLANT, AND CONDUCTOR MATERIAL

Core	Refrigerant	Capitalised costs of cable			
		I^2R loss	Plant and drive power	Conductor material	Approximate total
		£/km	£/km	£/km	£/km
Cu	—	3200	—	10000	13000
Al	H_2, N_2	17	21260	3000	24000
Be	N_2	62	5170	800000	800000
Nb	He, N_2	—	9203	3000	12000

3.8 Comment

Table 7 shows that only niobium has any hope of defraying its refrigeration costs by savings in conductor material. But its impracticably large core diameter and vulnerability to moderate faults present very serious objections to the use of a soft superconductor. Other superconductors are also ruled out. A thin-walled hard superconductor of 6cm diameter, if assumed capable of 10^6A/cm², would add 3W/km of electrical loss at 4°K when carrying 1600A r.m.s., but during a 7600MVA fault the temperature of the conductor would rise through 4degK (and so become dangerously unstable) in 0·18s. Such a hazard is clearly unacceptable.

While nothing can be certain about either the practicability or the costs of making and laying cables that have cores which are thermally insulated and refrigerated, these costs are certain to exceed by a considerable margin the corresponding figures for conventional cables. Consideration of operating expenses, of the vulnerability of the refrigerated network to extraneous damage, and of the hydrogen hazard, must also weigh in favour of conductors at normal temperature.

Had an overall improvement in cable economy seemed possible, that could be at all commensurate with the striking improvements in metal conductivity itself, there would be some reason for tackling the technical difficulties that appear. However, the naturally reduced penetration of alternating current into highly conducting material, and the acute intolerance of fault overloads by refrigerated conductors, together with the complexity and cost of refrigeration, remove all economic incentive for an attack upon the major problem of constructing and laying such cables.

4 Refrigerated transformer windings

The ambient magnetic fields in which transformer conductors must work are higher than in cables, and while there is relatively little magnetoresistive effect with aluminium, the effect becomes evident with beryllium. Work in France has shown that at 10kGs—which is the order of density to be expected under fault conditions—beryllium would increase in effective resistance 2·6 times. Also, taking into consideration that its forecast price is 1300francs/kg,[5] it is proposed in this inquiry simply to use the low-field resistivity of $2 \times 10^{-8}\Omega$cm as a guide to the influence of conductor resistivity on transformer design, and to consider the material itself not as beryllium but, for this purpose, as an imaginary metal.

In examining the prospects for using refrigerated aluminium at 20°K or metal with a resistivity of $2 \times 10^{-8}\Omega$cm at 77°K, as the conductor in transformers, it is necessary to resolve a number of questions. Their answers must decide whether metals with these resistivities should be used as wire or foil, and in what diameters or thicknesses, and whether their use in the preferred manner can show economic advantages, manifest either as reduced loss or in smaller overall size.

Because any change to the refrigerated form of transformer would be a major undertaking, its chance of economic success will be greater in the large power ratings. In this respect the analysis will follow the superconductor inquiry,[7] and present its findings in terms of a 570MVA generator transformer.

4.1 Losses in a wire-wound coil

The failure of current to remain uniform over the section of any conductor tends to be more marked in a transformer coil than it would be in a cable, because of the correspondingly greater ambient field. Fig. 9 presents the situation as experienced by a round wire or strand that is imagined

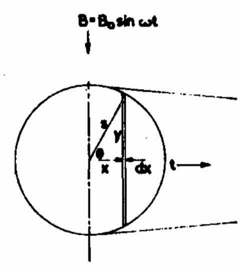

Fig. 9

Quantities used in determining eddy-current loss in one turn of wire strand of radius s

as part of one turn in a transformer coil. The wire radius s is deemed to be small compared with the penetration depth* that would correspond to the metal's resistivity. Each turn of wire is supposed to be carrying current of the same magnitude and phase as all its fellow wires, independently of whether they are in series or in parallel with it. B is the transformer leakage field density acting in a direction normal to the

* 700 $\rho^{1/2}$ cm at 50c/s

plane of the turns, and it is also in phase with the current in each turn.

If the current density that is attributable to uniform conduction is

$$j_0 = J_0 \sin \omega_t$$

j_0 must be regarded as modified by the addition of an eddy-current component j_e, so that the combined current density in the strand is

$$j = j_0 + j_e$$

where $j_e = (B_0\omega \cos \omega t \times 10^{-8})/\rho$

Now loss in the strip dx, for the length of one turn, can be written

$$dP = 2j^2\rho tydx$$

or

$$dP = 2(J_0 \sin \omega t + B_0 s\omega \cos \omega t \cos \theta/\rho \times 10^{+8})^2 \rho t s^2 \sin^2\theta d\theta$$

Integration leads to the loss in one turn

$$P_t = \frac{\pi s^2 t\rho J_0^2}{2}\left[1 + \left\{\frac{B_0\omega s \times 10^{-8}}{2\rho J_0}\right\}^2\right] \quad . \quad . \quad . \quad (19)$$

Fig. 10 shows a simple arrangement where the winding pair consists of primary and secondary coils that are uniformly wound, are of equal length, and have small radial build compared with the winding radius. Flux density at a radial position z is then

$$B_0 = \hat{B}z/b$$

where the peak flux density in the path between the coils is related approximately* to current density by

$$\hat{B} = 4\pi J_0 bk$$

The number of turns in the column dz is then

$$kldz/\pi s^2$$

* This neglects the excitation required at the ends of the coil

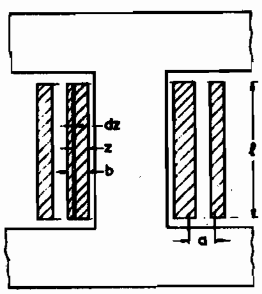

Fig. 10

Single transformer coil pair, defining quantities relating to eddy-current loss

By integrating eqn. 19, the combined conduction and eddy-current loss in the coil is found to be

$$P = kM\rho J_0^2(1 + h)/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where M is the gross volume of the coil, and the term representing eddy-current loss is given by

$$h = 13 \cdot 1 \times 10^{-18}(\omega sbk/\rho)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

At 50c/s eqn. 21 can be rewritten to give the strand radius

$$s = 0 \cdot 878 \times 10^6 \rho h^{1/2}/bk \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This result is illustrated, for a coil where the radial build b is 4cm and $k = 0 \cdot 5$, in Table 8.

In a conventional power transformer, h does not normally exceed 0·2; so it is evident that, by any standard, a major increase in manufacturing effort would be called for, both in wire drawing and in coil winding, to achieve a reduction in winding loss that is commensurate with the intrinsic drops in resistivity.

Eddy-current loss in a transformer winding is due to flux that penetrates below the surface of the conductors, and to reduce this loss effectively, wire must have an embarrassingly small gauge. But in a transformer where the leakage flux can be made to have substantially one direction, there is no greater freedom from eddy currents in wires than in foil of comparable gauge, if this is made to extend across the full width of a layer.

4.2 Losses in a foil winding

Fig. 11 indicates the foil dimensions that are relevant, and it can be seen that a slice of the foil dx thick that is distant x from the central 'plane' of the foil will carry a density of eddy current given by

$$j_e = B_0\omega \cos \omega t/\rho \times 10^8$$

So the density of conduction and eddy currents combined is

$$j = J_0 \sin \omega t + B_0\omega \cos \omega t/\rho \times 10^8 \quad . \quad . \quad . \quad (23)$$

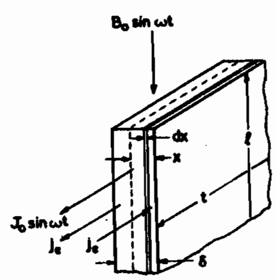

Fig. 11

Basis for calculating eddy-current loss in one turn of a foil-wound coil

l = width of layer
t = length of turn
δ = foil thickness

Table 8

SHOWING THE NECESSITY FOR A LARGE NUMBER OF SMALL-DIAMETER WIRES IF h, THE RATIO BETWEEN EDDY-CURRENT AND CONDUCTION LOSSES IN A WIRE-WOUND COIL, IS TO REMAIN SMALL

	$\rho = 2 \times 10^{-8}\Omega$cm			$\rho = 3 \times 10^{-9}\Omega$cm (Al)		
h	s	Wire dia.	Number of strands per cm of coil length	s	Wire dia.	Number of strands per cm of coil length
	cm × 10^{-3}	mil		cm × 10^{-3}	mil	
0·1	2·8	2·2	82000	0·42	0·33	3600000
1	8·8	6·9	8200	1·34	1·03	360000
10	28	22	820	4·2	3·30	36000

The corresponding combined loss in the volume $tldx$ is therefore

$$dP = \rho tl(J_0 \sin \omega t + x \cos \omega t B_0 \omega 10^{-8}/\rho)^2 dx$$

Integration, first over both halves of the foil and then with respect to time, gives the loss in one turn of foil:

$$P_t = \frac{tl\delta\rho J_0^2}{2}\left\{1 + \frac{1}{12}\left(\frac{B_0\omega\delta \times 10^{-8}}{\rho J_0}\right)^2\right\} \quad . \quad . \quad . \quad (24)$$

and in a coil of N turns for which flux density varies from zero to $\hat{B}$, where $\hat{B} = 0{\cdot}4\pi N\delta J_0$, loss for the whole coil, if its radial build is small, is approximately

$$P = N\delta tl\rho J_0^2(1 + h)/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where, at 50c/s, the eddy-current-loss term is

$$h = 0{\cdot}429 \times 10^{-12}(N^2\delta^4/\rho^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Because eddy-current loss varies with δ^4, it becomes evident that to wind a packet of n foils, that are connected in parallel at their ends but are not transposed, would have the effect of increasing h by the factor n^4, and by a still larger factor if foils are spaced apart in the packet. The proper transposition of wide foils would clearly prove a very difficult operation; so it would be natural, in meeting transformer-ratio requirements, to employ—for a low-voltage winding—singly wound coils connected in parallel, rather than multiple foils.

4.3 Foil thickness

It is now appropriate to decide optimum values for the thickness δ of single foils. Since there is to be only one foil in a coil, this can be done more conveniently in terms of the coil's r.m.s. current

$$I = J_0 l\delta/\sqrt{2}$$

so that

$$P = I^2\rho Nt(1 + h)/l\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

For a given current, number of turns, coil size and resistivity, the coil loss so far considered is least when the expression

$$(1 + h)/\delta$$

is a minimum, and it follows from eqn. 26 that this occurs when foil thickness has the optimum value

$$\delta_m = 934\left(\frac{\rho}{N}\right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

which corresponds to the value

$$h_m = 1/3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

4.4 Foil-edge losses

In wire or strip-wound coils the ampere-turns are positioned axially by the winding, but this is not the case with wide foils. A foil layer that is highly conductive acts as a boundary to alternating flux by its tendency to set up diamagnetic currents, and it is this feature that must now be considered. In the foil-wound pair the constraint of flux, and therefore the axial distribution of current per unit length, is uniform along most of the length marked l in Fig. 10, and the more conducting the foil and the closer the primary and secondary pair are to each other, the greater is the fraction of l that carries uniform current density. It is the ends of the coil, at the edges of the foil, where nonuniformity appears.

Nonuniformity, and its consequent extra losses, are invited if a foil winding is tapered in width in the manner conventionally used at high voltage. This is because edges of the wider foils would be called upon to exercise a flux constraint which, in the body of the coil, is less because it is there shared with fellow turns. For the purpose of this analysis, each foil layer will be assumed to be of uniform diameter and width; the rectangular ends of a pair of such coils are shown in Fig. 12.

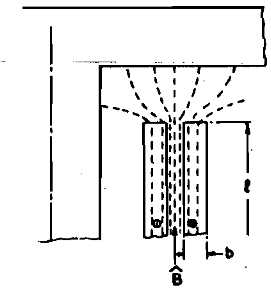

Fig. 12
Emergency of leakage flux from the uniform ends of a highly conducting foil-wound transformer pair

Even at ends that are rectangular there must be a concentration in current in order to support the leakage flux in its passage between leaving an intercoil space and entering either a neighbouring coil space or the yoke. In a foil winding this current increase is confined to the foil border and will be termed I_b. In order to form some estimate of the loss that would arise from I_b, it is helpful to picture the reason for its presence. This can be done by imagining in Fig. 13 two fine-wire* coils placed as extensions to the main ones, and

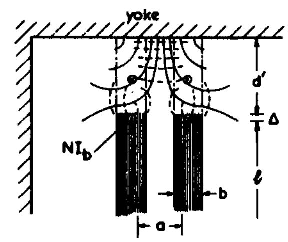

Fig. 13
Illustration of an estimation of foil-edge currents in the region Δ
The chain dotted lines enclose imaginary extension coils

energised so that magnetic excitation by the main pair merges without change into the extension, and continues up to some arbitrary yoke ceiling. This has served to remove I_b from the foils; to bring it back, imagine the main coils still in position but short-circuited so that neither links any flux. Then, with the extension coils excited in phase reverse, there will arise in the main foil edges m.m.f. equal to NI_b, in order to prevent flux from the extension coils passing into the main foil space as it would otherwise tend to do.

In Fig. 13, the axial length d' of the extension pair is shown greater (as it would require to be, to allow for end insulation and for the Dewar space) than the effective leakage-path width a. Under these conditions, it is seen that the m.m.f. NI_b is that necessary to neutralise an axial length of the extension coil that is of the order $a/2$; so that, as an approximation,

$$I_b = Ia/2l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Fig. 13 shows the edge current I_b within a narrow border of foil that is Δ deep. The distribution of I_b within the foil must result from penetration mechanisms that involve both attenuation and phase change with depth, but an approximate assessment of flux penetration across a series of foils, as

* Not foil, as foil would involve additional edge currents and so obscure the argument

distinct from a solid surface, can be deduced from Fig. 14. This depicts a radial field $B' \sin \omega t$ constrained at the edge of a foil winding of large diameter by means of boundary

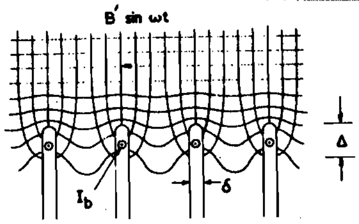

Fig. 14
Effect at foil edges
Field lines representing the contribution that must be made by edge currents I_b

currents I_b that flow near the foil edges; the resulting current density, if imagined constant over the distance Δ, would be

$$j_b = bB' \sin \omega t / 0{\cdot}4\pi\mu N\Delta\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The depth to which I_b effectively penetrates must be one at which its associated flux penetration would cause a second edge-current component in phase quadrature, but comparable in magnitude, with j_b. Now the electric field caused at the extreme foil edge by flux density that drops from $B' \sin \omega t$ to zero in a distance Δ is

$$0{\cdot}5 \times 10^{-8}(\Delta B'\omega \cos \omega t) \text{ volts/cm}$$

and the resistance-limited current density to which this would give rise at the extreme edge is

$$(10^{-8}\Delta B'\omega/2\rho) \cos \omega t \text{ ampere/cm}^2$$

so, from eqn. 31, the value of Δ at 50c/s approximates to

$$\Delta \simeq 0{\cdot}7 \times 10^3 \left(\frac{\rho b}{N\delta}\right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

From eqns. 30 and 32, the approximate loss for the two edges of one coil can be written

$$P_b = 7 \times 10^{-4}(a^2 N^{1/2} \delta^{1/2} / l \rho^{1/2} b^{1/2}) I^2 \rho N t / l\delta \quad . \quad (33$$

(the term beyond the right-hand bracket is ohmic, or conduction loss).

The three loss components in each coil can now be collectively expressed as

$$P_c = (I^2 \rho N t / l)(\delta^{-1} + 0{\cdot}429 \times 10^{-12}\delta^3 N^2/\rho^2 + 7 \times 10^{-4}\delta^{-1/2} a^2 N^{1/2} / l \rho^{1/2} b^{1/2}) \quad (34)$$

Here, the three terms in δ represent, respectively, conduction, eddy-current, and edge losses in one coil that is wound with plain foil.

4.5 Coil design for a 570MVA transformer

To reduce conductor loss requires a high core flux; so, in considering a winding design, the core will be assumed to be as large in section as would normally be used, and the winding contrived in as small a bulk as the conductor and insulation will allow.

In deciding the minimum-bulk design for a 570MVA, 400/22kV generator–transformer employing ideal superconductive film,[7] multiple-coil pairs were considered with coils in parallel on the l.v. side and in series on the h.v. side, arranged to give four axial paths for leakage flux.

In this paper, the transformer core will be assumed to have a section of 11600cm² and to work at 15·5kGs peak. Quantities pertaining to eqn. 34 are listed in Table 9.

In Fig. 15 the separate components of conduction, eddy-current, and edge loss are plotted against foil thickness for a single-path design with a coil length of 280cm, which is about that of a conventional winding. The curves show the influence of edge loss in deciding optimum foil thickness, and they indicate the relatively high incidence of this loss, particularly in the l.v. coil with its fewer turns. Fig. 16 shows corresponding curves for a 4-path winding which is one-quarter as long.

Table 9

QUANTITIES AFFECTING CONDUCTOR LOSS IN A 570MVA GENERATOR–TRANSFORMER DESIGN

	Single path	Four paths
Coil current I_1, A r.m.s.	8620	2160
I_2, A r.m.s.	820	820
Coil turns N_1	55	55
N_2	580	145
Mean length of turn t_1, cm	453	540
t_2, cm	500	540
Effective radial width of flux path, a, cm	6	6
Coil radial build b_1, cm	0·83	0·71
b_2, cm	5·8	1·59

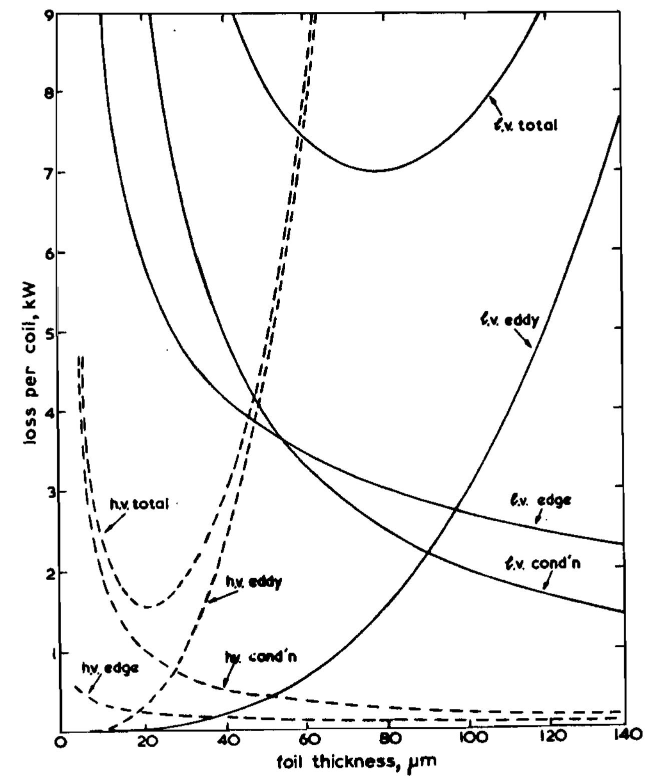

Fig. 15
Variation with foil thickness of conduction (ohmic), eddy-current and edge loss in a single pair of foil-wound coils
Single flux path, 280cm long

This would give a much smaller transformer height, but the level of loss is such that refrigerator drive power would itself exceed the I^2R loss in a conventional design.

The indication from eqn. 34 is that winding length must increase if losses are to be reduced, and to minimise the heavy edge-loss component, the effective flux-path width a must remain as small as the requirements of insulation and of cooling will allow. By restoring winding length to 280cm for a 4-path assembly, coil losses become reduced to 10·2kW, and at this level they would call for 398k Win the refrigerator drive.

It will be evident from the nature of edge currents that any prominent foil edge must undergo a greater flux penetration than its neighbours, and that this must lead to local current reversals and greater general edge loss than is defined by eqn. 33. To avoid this, care would be necessary to ensure an alignment of foil edges that is closer than the penetration distance Δ of about 500μm (20mil). A design which, in principle, reduces edge losses will now be examined.

4.6 Curled foil edges

An effective electromagnetic method of reducing edge losses lies in curling the edges of foils as shown in Fig. 17 and described in Reference 7. If it were feasible to carry out this potentially difficult manufacturing process the result could reduce edge-current density to a level that is about three times the coil-body density and, moreover, spread it over a curved length of foil commensurate with $b/2$; so that, approximately,

$$I_{edge} = 3Ib/l$$

but the resistance of this band of foil is

$$2N\rho t/b\delta$$

so the loss from both curled edges of a coil is approximately

$$P_r = (I^2\rho Nt/l)(9b/l)\delta^{-1}$$

Eqn. 34 can therefore be rewritten for curled foils as

$$P_{cr} = (I^2\rho Nt/l)\{\delta^{-1}(1 + 9b/l) + 0{\cdot}429 \times 10^{-12}\delta^3N^2/\rho^2\} \quad (35)$$

Using eqns. 34 and 35, losses have been assessed at 570MVA for a number of designs, all with the same core area of 11600cm^2, but with coil lengths varying from a conventional 280cm to one quarter of this length. The effect of a change from plain to curled foils, and of a change in resistivity from 3×10^{-9}ohmcm for aluminium at 20°K to $2 \times 10^{-8}\Omega$cm at 77°K, are included in the summarised results of Table 10.

4.7 Transient thermal loads

A power transformer must withstand system faults whose severity can range from a relatively mild 3·3 times increase in current, in the case of a generator–transformer where fault is limited by the generator impedance, to over 20 times rated current in a substation transformer. It is necessary both to carry such faults for periods of 1–2s and to tolerate autoreclosing; so the total duration at any one event could amount to three or more seconds. Of the aluminium-foil designs listed in Table 10, the 280cm 4-path curled-edge winding offers the best saving in energy, with a conductor loss of 5·73kW at rated load. In this design the greatest current density occurs at the curled edges of the l.v. coils, and calculation shows that during a fault which lasts 4s at 3·3 times current the l.v. coil edges would rise from

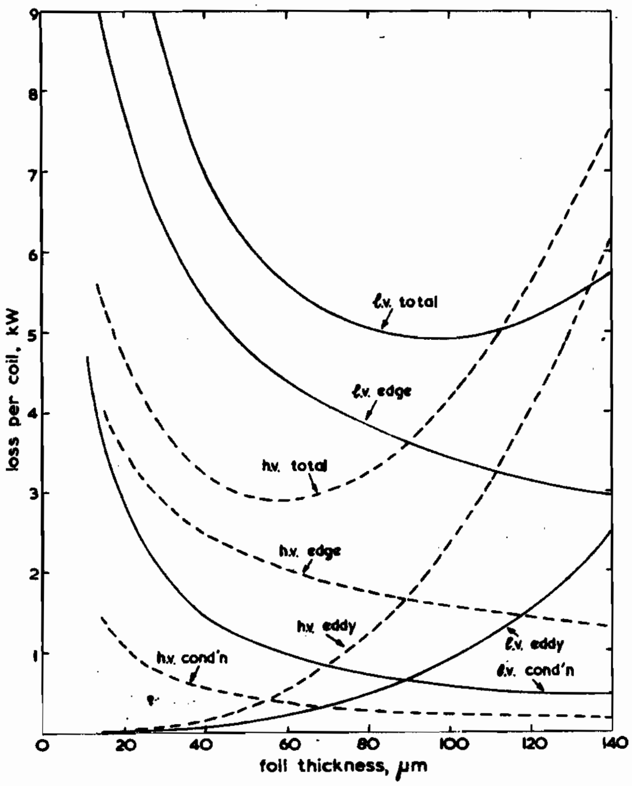

Fig. 16
Effect of foil thickness on losses for a 4-path coil design 70cm long

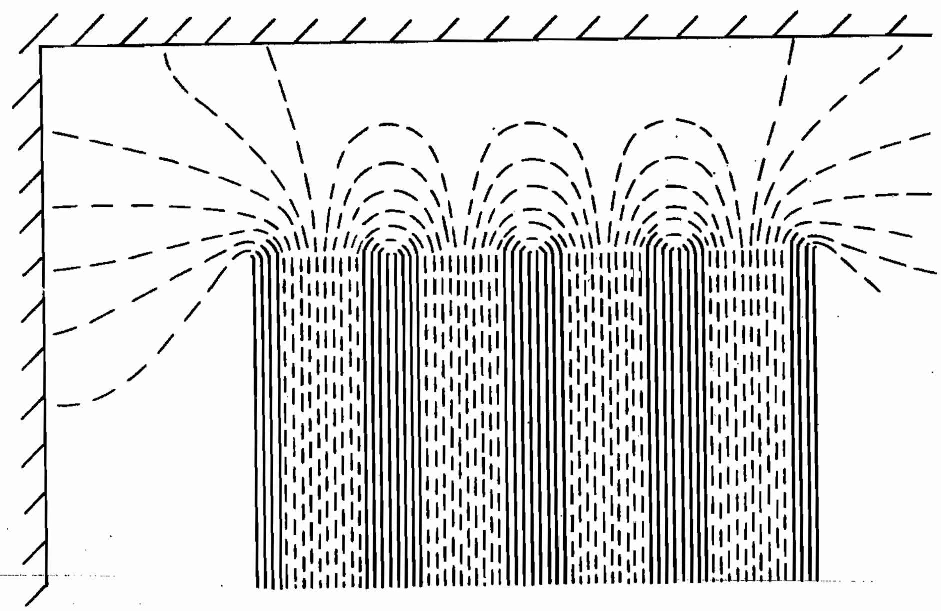

Fig. 17
Radial cross-section showing the leakage-flux field near the ends of a foil-wound 4-path coil system
Foil edges are curled in a way to minimise loss from edge currents

Table 10

A COMPARISON OF CONDUCTOR LOSSES AND REFRIGERATOR POWER AT TWO RESISTIVITIES, OVER A RANGE OF WINDING LENGTH

Resistivity and temperature	Foil edges	Number of flux paths	Coil length	Reactance	Optimum values					
					Foil thickness		Coil loss		Total of all coils	Pump power
					l.v.	h.v.	l.v.	h.v.		
			cm	%	μm	μm	kW	kW	kW	kW
$3 \times 10^{-9}\,\Omega$cm 20°K (Aluminium)	plain	1	280	4·77	80	22	6·98	1·58	25·7	1000
		4	280	1·34	78	50	0·53	0·32	10·2	398
		4	140	2·68	85	52	1·56	0·94	30·0	1170
		4	70	5·36	96	58	4·90	2·90	93·5	3650
	curled	4	280	1·34	70	45	0·291	0·186	5·73	223
		4	140	2·68	70	45	0·591	0·387	11·7	456
		4	70	5·36	72	45	1·22	0·834	24·6	960
$2 \times 10^{-8}\,\Omega$cm 77°K	plain	4	280	1·4	200	120	1·31	0·81	25·4	182
		4	140	2·8	200	120	2·61	1·69	51·5	382
		4	70	5·6	220	130	7·28	4·55	142	1050

20° K to 44° K. This would cause coil-edge resistivity to rise tenfold; so, with resumption of normal load after the fault, edge-loss density would momentarily be ten times greater than before. The situation is acutely worse with higher overloads. Table 11 shows the way in which loss density and temperature of the l.v. curled edges would both rise with fault duration if the fault were at 10 times rated current. There is now clearly a danger of thermal instability when full-load current is resumed, for at the end of 1·7s the l.v. edges would have risen to 160° K, and, on resuming normal load, edge dissipation at this temperature would be about 10 W/cm^3, or 1470 W for the curled edges of each l.v. coil. Losses in the h.v. coils and in the body of the l.v. coils will also increase, though to a smaller extent, and the effect of 1·7s of a tenfold current overload is estimated to raise the conductor loss in this transformer from 5·73 kW to 26 kW at normal current, with 18 kW in the l.v. edges. The transformer would therefore prove thermally unstable after this fault unless its refrigeration, notably of the l.v. coil edges, could respond at these higher rates.

Table 11

THE RISE OF ALUMINIUM TEMPERATURE AND LOSS DENSITY IN THE CURLED EDGES OF LOW-VOLTAGE COILS WHEN CARRYING TEN TIMES RATED CURRENT (COIL LENGTH 280 CM, FOUR-PATH DESIGN)

Fault duration at 10 times rated current	Loss density	Temperature
s	W/cm^3	°K
0	0·031	20
1·0	3·06	45
1·2	6·30	60
1·4	16·4	82
1·5	20	96
1·6	38	122
1·7	70	160
1·8	100	204

4.8 Discussion

This analysis has been limited to broad questions that could affect the behaviour of high-conductivity windings in transformers. It has not considered such problems as foil terminations, tappings, bushings, the support of coils against heavy forces, the construction of electrically open Dewar systems, or the design of cooling ducts able to operate over wide ranges of temperature and pressure. Consequently, the findings are limited and will certainly be optimistic. If I^2R loss in a conventional 570 MVA design is taken to be 2 MW, aluminium foil at 20° K could, in principle, lead to a saving of 1750 kW or, if capitalised at £145/kW, £254000. To do this, however, the winding length along the transformer core would have to be 280 cm, and the transformer dimensions would be no more favourable than those of a conventional one. On the debit side, hydrogen refrigeration plant, if rated for 20 kW, would cost about £190000, but this does not include any sum for installing, housing and supervising the plant or provision to protect against the hazards associated with liquid hydrogen. If there were a conductor that possessed a resistivity of $2 \times 10^{-8}\,\Omega$cm at 77° K when in high magnetic fields, and that did not suffer from beryllium's high cost disadvantage, it would show a marginal improvement over aluminium at 20° K, but not sufficient to make the proposition attractive.

It must be concluded that for transformers, as for cables, there is insufficient economic reason for seeking to use highly vulnerable deeply refrigerated windings.

5 Acknowledgments

This work was inspired by studies made in France by P. Burnier and others concerning the application of refrigerated aluminium and beryllium. It has been helped by discussions with colleagues in the transformer and Cable Divisions of AEI Ltd., with Dr. D. A. Parker on the subject of viscous flow, and with the British Oxygen Co. on the likely future performance and cost of refrigeration plant. The author's thanks are also due to Dr. J. E. Stanworth, Director of Central Research, AEI Ltd., for his active support at all stages of the work.

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7 Appendix

Loss in stranded cables

Fig. 6 indicates an annular section containing N insulated strands, each with radius s, that share current equally. They are arranged with the same number n in every layer; so, if insulation thickness is ignored,

$$2\pi r = 2Ns/n$$

and $\alpha r = 2ns$

whence

$$N = \pi r^2\alpha/2s^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If the current that is productively carried by the core is

$$I = I_0 \sin \omega t$$

the amount which lies within a radius $(r + x)$ is

$$Ix/\alpha r$$

this will give rise to a circumferential flux density that is in phase with I, and given by

$$B_0 = 0{\cdot}2I_0\mu x/(r + x)\alpha r$$

By an argument similar to that which led to eqn. 19, it can be shown that the element of loss in the band dx (Fig. 6) is

$$dP = (I_0^2\rho/\pi^2r^3\alpha^2)[1 + \{(\pi^2\mu\omega 10^{-9}sr/2\rho)x/(r + x)\}^2]dx$$

Integrating the term in x between the limits of 0 and αr leads to

$$r\{\alpha(2 + \alpha)/(1 + \alpha) - 2 \log_e (1 + \alpha)\}$$

So, by expressing r in terms of R and α, the loss per centimetre of the whole core can be written

$$P = I_0^2\rho(E + F)/\pi^2$$

where E and F have the values defined by eqns. 16, 17 and 18.

Discussion on

Method of predicting the thermal loading of an oil circuit breaker

Mr. W. Watson: The title of the paper would be more explicit if it were modified to 'a particular design of oil circuit breaker'. It is not brought out in the paper that the circuit-breaker design tested must have been of three separate single-phase tanks, which is not a particularly common arrangement at the present time. This design lends itself to the relatively simple analogue analysis detailed in the paper in a much better way than indoor metalclad gear, with its relatively small phase centres, associated proximity effects, isolating features, appreciably higher current ratings than those considered in the paper, and the greater probability of considerable heat flow from one part or phase of the circuit breaker to another.

The authors have gone to great trouble to predict the temperature rises at various points along the insulated bushing conductor, but the much more important contact region has been ignored, which is to be regretted, since the analogue could have been arranged to examine this portion also. Information on bushing-conductor temperature rise is very necessary for design purposes; it can be estimated fairly simply, based on measurements of the hot d.c. resistance of the conductor and of its end temperatures.

The predictions in the paper also rely on measurements of oil and conductor-end temperatures; these are not available if a new design is being assessed. The oil-temperature rise can, however, be reasonably estimated; for instance, using the authors' figures, the temperature rise above ambient of the tank is about 21degC, and the temperature rise of the oil is 19degC above that of the tank; i.e. a 40degC rise above ambient, which is in agreement with the measurements indicated in Fig. 1. This estimate makes no allowance for heat flow into the bushings, and it confirms the authors' assumption that the contact loss is dissipated in the oil.

The temperature-rise figures quoted in Fig. 1 for unplated contacts, i.e. 40 and 45degC, are considered too high for thermal stability, and, if the breaker is identical with respect to both poles, the difference of 5degC could be taken as an indication of contact deterioration. If this is so, the suggestion in the paper of higher continuous-current ratings for the breaker, based on these and other figures, is hardly justified.

It must be remembered that the British Standards contain temperature-rise figures based on straightforward measurements at accessible parts, using thermometers or thermocouples, the figures being backed by years of service experience. It follows that these figures must take some account of the hidden temperature rises of the type investigated by the authors. Bearing these facts in mind, the overzealous application of the figures in the British Standards, together with the more refined methods of measurement possible today, can cramp the style of the switchgear designer, and this could result in poor designs and the uneconomic use of copper, particularly with the high current ratings required in present-day sizes of switchgear. It is of great importance that there are proposals in international specifications that temperature rises considerably in excess of the present accepted figures should be allowed for, in certain contact and conductor arrangements. It is to be hoped that these figures will be embodied in the British Standards in the foreseeable future.

Mr. H. G. Bonson: The paper rightly draws attention to the important role which contact resistance plays in determining the temperature rise of a circuit breaker. Table A

Table A

132/33 kV GRID TRANSFORMERS AND ASSOCIATED 33 kV-SWITCHGEAR RATINGS

BS 171: 1959 rating		Cyclic rating (130%)	BS 116: 1952 ratings available	Variation of current rating with contact resistance
Apparent power	Current at 33 kV			
MVA	A	A	A	A
45	787	1020	800 1200†	800–1000 1280–1500
60	1050	1370	1200† 1600	1280–1500 1700–2000
90	1575	2050	1600 2000*†	1700–2000 2130–2500

* 2000 A is not a preferred rating at 33 kV; but it is at lower voltages, and it is readily available at 33 kV
† Rating used for transformer listed in first column

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