fields. In the case of integer spin the right hand side of (7) must be slightly more complicated to take the auxiliary conditions into account.

*V. Vanyashin's paper "Second-Order Wave Equations for Spinor Wave Functions" at the Conference on the Theory of Elementary Particles, Uzhgorod, October 1958.

[1] J. Schwinger, Phys. Rev. 91, 713 (1953).

Translated by D. ter Haar
384

COMPARISON OF THE MACROSCOPIC THEORY OF SUPERCONDUCTIVITY WITH EXPERIMENTAL DATA

V. L. GINZBURG

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor February 19, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 36, 1930-1932 (June, 1959)

GOR'KOV[1] has recently shown that the macroscopic equations for superconductors, established earlier by Landau and the author[2] (see also references 3 and 4) follow from the current microscopic theory of superconductivity. He obtained then an essentially new result, namely a confirmation that the charge e_{eff} which occurs in these equations is equal to twice the electronic charge, 2e. This result has an obvious physical meaning since the charge of a Cooper pair is just equal to 2e. Meanwhile, the charge e_{eff} was previously usually put equal to e. It is in that connection advisable to consider a comparison of the macroscopic theory with experiments, putting $e_{eff} = 2e$. The parameter κ entering into the theory is then equal to

$$\varkappa = \frac{\sqrt{2}\,|e_{eff}|}{\hbar c} H_{cM}\delta_L^2 = 4.32\cdot 10^7 H_{cM}\delta_L^2, \qquad (1)$$

where H_{cM} is the critical magnetic field and δ_L the depth of the penetration of the field in a bulk metal at the given temperature T. It is now essential that the theory of reference 2 in a weak field goes over into the theory of F. and H. London and that δ_L in (1) is thus the London penetration depth. Near the critical temperature (this will be the only region with which we shall be concerned) the measured penetration depth δ is for all metals equal to δ_L. If, however, for tin $\delta \approx \delta_L$ for $\Delta T = T_c - T \lesssim 0.1°$ with an accuracy of 10 to 15%, then for aluminum, for instance, $\delta \approx \delta_L$ only when $\Delta T \lesssim 10^{-3\circ}$. As a result one can for tin, lead, and some other superconductors (in contradistinction to aluminum) determine the value of κ near T_c directly from the experimental data for H_{cM} and δ. Such a method is very suitable since Eq. (1) is practically independent of any assumption when $\delta = \delta_L$ and $T \rightarrow T_c$ (the result $e_{eff} = 2e$ was obtained for an isotropic model[5] but is most probably much more generally true).

If we use the empirical law

$$\delta = \delta_{00}\,[1 - (T/T_c)^4]^{-1/2},$$

we have near T_c

$$\delta = \frac{1}{2}\delta_{00}\sqrt{\frac{T_c}{\Delta T}} \qquad H_{cM} = \left|\frac{dH_{cM}}{dT}\right|_c \Delta T,$$

$$\varkappa = 1.08\cdot 10^7 \left|\frac{dH_{cM}}{dT}\right|_c T_c\delta_{00}^2.$$

For tin

$$\left(T_c = 3.73°,\ \left|\frac{dH_{cM}}{dT}\right|_c = 151, \quad \delta_{00} = 5.1\cdot 10^{-6}\,\text{cm}\right)$$

we have thus $\kappa = 0.158$. The limiting field for supercooling H_{ci} is for such a value of κ equal to $H_{c1}/H_{cM} = \sqrt{2\kappa} = 0.224$. Experimentally[6] $H_{c1}/H_{cM} = 0.232$. For the surface energy $\sigma_{ns} = H_{cM}^2\Delta/8\pi$ we have[7] for $\kappa = 0.158$

$$\Delta = 6.5\delta_L \approx 1.66\cdot 10^{-5}\sqrt{T_c/(T_c - T)}$$

while we have experimentally, instead of 1.66, according to Sharvin's data[8] 2.5 and according to Faber's data[9] 1.88. Since in both cases $T_c - T > 0.1°$ and we are dealing with a limiting law as $T \rightarrow T_c$ we can as yet scarcely consider the discrepancy obtained here to be real (if we determine κ from Faber's data for Δ we get $\kappa = 0.15$ and $H_{c1}/H_{cM} = 0.212$). In the isotropic model[5] near T_c

$$\delta_L(T) = \delta_L(0)\sqrt{T_c/2\Delta T}, \quad \delta_L^2(0) = mc^2/4\pi e^2 n \qquad (2)$$

(n is the concentration of "free electrons"). If we use this expression, Eq. (1) takes the form

$$\varkappa = 2.16\cdot 10^7\,|dH_{cM}/dT|_c\,T_c\delta_L^2(0). \qquad (3)$$

For tin $\delta_L(0) = 3.5\times 10^{-6}$, according to references 10 and 6, whence $\kappa = 0.149$. The value $\kappa = 0.15$ to 0.16 for tin agrees thus with sufficient accuracy both with experiments and with the requirements of the macroscopic as well as of the microscopic theory.

A further check must, in particular, consist in the measurement of a third effect: the change of δ with field.[2,7] The increase of δ in tin near T_c

for an external constant field equal to $H_0 = H_{cM}$ must be $3\kappa/4\sqrt{2} = 8.5\%$ (when we measure in a weak variable field parallel to H_0) and $\kappa/4\sqrt{2} = 2.8\%$ (when we measure in a weak field perpendicular to H_0). We note also that a sharp change in the behavior of Sn + In alloys takes place for 2.3% In (see references 11 and 12) when $\kappa \approx 0.6$ since, according to Chambers,[13] the penetration depth approximately doubles then. Theoretically, however, the change in the properties must occur[2,14] for $\kappa = 1/\sqrt{2} = 0.707$. The well known vagueness particular to experiments on alloys makes it hardly possible to speak here about a discrepancy between theory and experiments. In any case, the agreement is appreciably better for $e_{\mathrm{eff}} = 2e$ than for $e_{\mathrm{eff}} = e$.

If for Al we use in (1) the experimental value $\delta = \frac{1}{2}\delta_{00}\sqrt{T_c/\Delta T}$ with $\delta_{00} = 4.93 \times 10^{-6}$ (and also $T_c = 1.17$, $|dH_{cM}/dT|_c = 164$) we get $\kappa = 0.05$. In that case, however, $\delta \neq \delta_L$ unless we deal with values $\Delta T \lesssim 10^{-3}{}^\circ$. We must therefore proceed by two other ways. First we can determine κ through the equation[14] $H_{c1}/H_{cM} = \sqrt{2}\,\kappa$ from the experimental value[6] $H_{c1}/H_{cM} = 0.0363$. Hence $\kappa = 0.0256$ and, according to reference 7 and Eq. (2)

$$\Delta \approx 62\delta_L = 44\delta_L(0)\sqrt{T_c/\Delta T} = 10.9\cdot 10^{-5}\sqrt{T_c/\Delta T},$$

since we have $\delta_L(0) = 2.48 \times 10^{-6}$ if we use (3). Experimentally[9] $\Delta = 9.0 \times 10^{-5}\sqrt{T_c/\Delta T}$ for aluminum, i.e., we get excellent agreement with theory.* Second, we should determine $\delta_L(0)$ by a less consistent method of comparison from independent data; we make then additional assumptions and, according to references 10 and 5, we have for aluminum $\delta_L(0) = 1.6 \times 10^{-6}$ and from Eq. (3) $\kappa = 1.06 \times 10^{-3}$. Hence $H_{c1}/H_{cM} = 0.015$ and $\Delta = 18 \times 10^{-5}\sqrt{T_c/\Delta T}$ which disagrees with experiments by approximately a factor two. Since for aluminum and for other "Pippard" superconductors a comparison of the theory of reference 2 with experiments is difficult it is desirable for this purpose to use in the first instance "London" superconductors (at least for $\Delta T \lesssim 0.1°$) and first of all lead.

We note in conclusion that it leads to difficulties to use data on specimens of small dimensions (films and so on) for a direct comparison of the theory with experiments. This is connected with the polycrystalline structure of such specimens which leads to the impossibility to consider them to be equivalent, thin specimens of the same metal in bulk, but in single crystal form. One should therefore in accordance with the well known considerations of Pippard and others consider thin polycrystalline films rather like alloys.

*We must note that the region of applicability of the theory of reference 2 is considerably wider[4] if we evaluate the values of Δ and H_{c1} than if we evaluate δ. We shall therefore for Al use the necessary formulae of the theory of reference 2 also for $\Delta T \sim 0.1°$ when

$$\delta_L/\varkappa \sim 6 \times 10^{-6}/0.0256 = 2.3\cdot 10^{-4} > \xi_0 \sim 10^{-4} \text{ (see references 5 and 10).}$$

[1] L. P. Gor'kov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1918 (1959), Soviet Phys. JETP, this issue, p. 1364.

[2] V. L. Ginzburg and L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 1064 (1950).

[3] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 748 (1955), Soviet Phys. JETP **2**, 589 (1956).

[4] V. L. Ginzburg, Physica **24**, S42 (1958).

[5] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).

[6] T. E. Faber, Proc. Roy. Soc. (London) **A241**, 531 (1957).

[7] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 593 (1956), Soviet Phys. JETP **3**, 621 (1956).

[8] Yu. V. Sharvin, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1341 (1957), Soviet Phys. JETP **6**, 1031 (1958).

[9] T. E. Faber, Proc. Roy. Soc. (London) **A248**, 460 (1958).

[10] T. E. Faber and A. B. Pippard, Proc. Roy. Soc. (London) **A231**, 336 (1955).

[11] A. B. Pippard, Trans. Roy. Soc. (London) **A248**, 97 (1955).

[12] P. R. Doidge, Trans. Roy. Soc. (London) **A248**, 553 (1956).

[13] R. G. Chambers, Proc. Cambridge Phil. Soc. **52**, 363 (1956).

[14] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 541 (1956), Soviet Phys. JETP **4**, 594 (1957).

Translated by D. ter Haar
385