The Evolution of the Theory of Electron-Phonon Induced Pairing and its Limits for Superconducting Systems

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1911 THE FIRST SUPERCONDUCTOR



H. KAMERLINGH ONNES



CLASSES OF SUPERCONDUCTORS

CLASS 1: conventional metals, C60, some organics, doped semiconductors, MgB2

CLASS 2: copper oxides, heavy fermion metals, some organics



BCS Model (1957)
$$T_c = 1.13 T_D e^{-1/NV}$$

$$NV = \lambda - \mu > 0$$

T

1

attractive phonon interaction

repulsive Coulomb interaction

Improvements (1960's)

$$\lambda \to \lambda^* = \frac{\lambda}{1+\lambda} \qquad \mu \to \mu^* = \frac{\mu}{1+\mu \ln \frac{E_F}{E_D}}$$

CAN BCS THEORY PREDICT T.? $T_c \stackrel{o}{\longrightarrow} T_D \stackrel{i}{\leftarrow} \stackrel{i}{\longrightarrow} \gamma \sim IIK$ $300K \quad 0.3$ $I_F NV \stackrel{*}{\longrightarrow} 0.03 \quad T_c \rightarrow 10^{-12}$

NEED TO KNOW NV VERY ACCURATELY

DOPED SEMICONDUCTOR Sr Ti O3 FIRST SUPERCONDUCTING OXIDE (COHEN 1963; SCHOOLEY ET AL 1964)

CONCEPTUAL BASIS

ONE CAN ARGUE FOR TWO MODELS OR "MENTAL PICTURES" OF A SOLID:

1. INTERACTING ATOMS MODEL (REDUCTIONISM)

2. ELEMENTARY EXCITATIONS MODEL (EMERGENCE)

Standard Model Plane Wave Pseudopotential Method [PWPM]



Plane Wave Pseudopotential Method (Standard Model of Solids)

For a broad class of solids, clusters, and molecules, this method describes ground-state and excited-state properties such as: electronic structure crystal structure and structural transitions structural and mechanical properties vibrational properties electron-lattice interactions superconductivity optical properties photoemission properties

sh and hop SILICON To (PRESSURE)



THEORY

CHANG, DACOROGN A& COHEN

EXP. GRENOBLE : MIGNOT, CHOUTEAN + MARTINEZ BERKELEY! ERSKINE & YU

Calculational Methods



• Band structure



• Fermi surface consists of four sheets: two from σ -bonding boron $p_{x,y}$ orbitals and two from π -bonding boron p_z orbitals.

Also: An & Pickett, PRL 2001; Kortus, et al, PRL 2001; Liu, et al, PRL 2001; Kong, et al, PRB 2001; ...



Main ingredient: momentum- and frequencydependent Eliashberg function

$$\alpha^{2}F(\vec{k},\vec{k}',\omega) \equiv N(\varepsilon_{F})\sum_{j}\left|g_{\vec{k}\vec{k}'}^{j}\right|^{2}\delta(\omega-\omega_{j\vec{q}})$$

where $N(\varepsilon_F)$ = density of states per spin at Fermi level

g_{kk} = electron-phonon matrix element ω_{jq} = frequency of phonon in jth branch Equivalently:

$$\lambda(\vec{k},\vec{k}',n) = \int_0^\infty d\omega \alpha^2 F(\vec{k},\vec{k}',\omega) \frac{2\omega}{\omega^2 + (2n\pi T)^2}$$

 $\lambda = \langle \lambda(\mathbf{k}, \, \mathbf{k}', \, \mathbf{0}) \,$

Transition Temperature and Isotope Effect

	harmonic		anharmonic		experiment
	isotropic	anisotropic	isotropic	anisotropic	
T_c	28 K	55 K	19 K	39 K	39 K
α_B	0.42	0.46	0.25	0.32	0.26,0.30
$lpha_{Mg}$	0.04	0.02	0.05	0.03	0.02
λ	0.73		0.61		0.58, 0.62
ω_{ph}	62.7 meV		75.9 meV		75.9, 76.9

 $\mu^*(\omega_c) = 0.12.$

Anharmonicity --> smal

- λ : averaged electron-phonon coupling.
- ω_{ph} : frequency of the in-plane B–B stretching modes (E_{2g}) at Γ .

For $0.10 \leq \mu^*(\omega_c) \leq 0.14$, 41 K $\geq T_c \geq$ 37K



Notes: 1) most metals: $\lambda \sim 0.3 - 0.5$ 2) MgB₂: $\langle \lambda \rangle = 0.61$; specific heat data $\lambda = 0.58$, 0.62

Superconducting Gap at 4K Г Γ M M ρ(Δ) 2 6 0 4 8 Δ (meV)

- 🕰 (k) on Fermi surface at T=4 K
- Large gap on cylindrical σ–sheets
- 2 dominant sets of gap values

Specific Heat of MgB₂



Summary

- MgB₂ is a **multi-gap**, **phonon-mediated** superconductor
- Large electron-phonon coupling of the ${\bf \sigma}$ boron states responsible for high T_c
- Need to solve the **k-dependent** Eliashberg equations to obtain the correct $T_{\rm c}$ and other quantities
- First-principles results explained T_c , specific heat, isotope exponents, photoemission, tunneling, and other data
- Theory predicts at least **2 dominant** superconducting gap values at low temperature, and the two-gap feature is robust against pressure, doping and impurity scattering.

RAISING Tc

WE TRIED TO USE THEORY TO SUGGEST HOW TO INCREASE THE TRANSITION TEMPERATURE OF MAGNESIUM DIBORIDE SIGNIFICANTLY BUT FAILED!

THIS RESULT IS CONSISTENT WITH EXPERIMENTS UP TO NOW.

Phonon softening in superconductors



H. G. Smith and W. Glaser, PRL 25, 1611 (1970).

L. Pinstchovius et al., PRL 54, 1260 (1985)

L. Pinstchovius et al. PRB 28, 5866 (1983)

L. Pintschovius, phys. stat. sol. (b) 242, 30 (2005)

E_{2g} phonon in MgB_2 and AlB_2



 E_{2g} phonon in MgB_2 and AlB_2



K. P. Bohnen, R. Heid, and B. Renker, PRL 86, 5771 (2001).



SOLID C60

STANDARD MODEL

M3C60 = C60'S + SEA OF ELECTRONS



ELECTRON - PHONON SCATTERING





FIG. 4. Photoemission spectra of Rb_3C_{60} at 33 K (squares) and at 6 K (circles). The data are modeled (solid lines) with, respectively, a Fermi-Dirac function at 33 K and a BCS function with a gap $\Delta = 4.4$ meV, broadened by the resolution functions found for the Pt reference.

(SAME FOR K3 C60; D= 2.9 meV) HESPER ET AL

WE FIND THAT FOR C60 THAT THE ELECTRON -PHONON PARAMETER IN BCS $\lambda \text{-}\text{NV}$

THEREFORE TO GET HIGHER Tc's, WE :

CAN INCREASE "N" WITH CARRIERS BY DOPING OR BAND STRUCTURE [e.g. S. SAITO USING d-ELECTRON EFFECTS] OR

CAN INCREASE "V" BY INCREASING THE "CURVATURE" GRAPHITE \rightarrow NANOTUBE \rightarrow C60 \rightarrow C36 \rightarrow ?



Electron-Phonon Coupling and Superconductivity in Carbon Nanotubes THEORY: [L. X. Benedict, V. H. Crespi, S. G. Louie and M. L. Cohen, PRB

. X. Benedict, V. H. Crespi, S. G. Louie and M. L. Cohen, PR 52, 14935 (1995)]



- Large curvature of small diameter tubes leads to enhanced λ .
- Tc can be in the range of 10-20 degrees Kevin for diameter d < 5 A.

EXP: Z.K. TANG Etg!, Science 292, 2462 (2001)

K IN NANOTUBE



BN/C₆₀ Peapods





T_c formulas

$$T_c = 1.14\omega_{ph} \exp\left(\frac{-1}{N(0)V}\right)$$

BCS, 1957

$$T_{c} = \frac{T_{D}}{1.45} \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^{*}(1+0.62\lambda)}\right)$$

McMillan, 1968

1975

$$T_{c} = \frac{\left\langle \omega \right\rangle_{\log}}{1.20} \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^{*}(1+0.62\lambda)}\right) \text{ for } \lambda < 1.5$$

$$T_{c} = 0.183 \sqrt{\lambda \left\langle \omega^{2} \right\rangle} \text{ for } \lambda > 10, \ \mu^{*} = 0$$
 Allen and Dynes,

$$\lambda \equiv 2 \int_0^{\omega_{\text{max}}} \alpha^2 F(\omega) \omega^{-1} d\omega$$

+ anisotropic electrons, anharmonic phonons, etc...

ELECTRON-PHONON COUPLING

$$\lambda \left\langle \omega^2 \right\rangle = \sum_i \frac{\eta_i}{M_i}$$

SO λ CAN BE VIEWED AS THE RATIO OF AN ELECTRONIC SPRING CONSTANT η and a lattice Spring constant

Numerical results



Values of η

	η (eV/Ų)	$0.183\sqrt{\lambda\left\langle \omega^{2} ight angle }$ (K)	EXP
C (diamond)*	54	290	~ 10
C (graphite)*	48	270	?
BN*	36	240	?
Si*	10	82	~10

*at peak of $\eta(E)$

Strong coupling limit

 $T_c \leq 0.183 \sqrt{\lambda \langle \omega^2 \rangle}$

electron-phonon coupling strength Or the electronic spring constant /ionic mass



η / Μ

 $T_{c} \leq \alpha \sqrt{\Omega_{ph}^{2} - \omega_{ph}^{2}} \leq \alpha \Omega_{ph}$ stability of bare lattice


BEYOND BCS MODEL (FREQUENCY DEPENDENCE) A, V, N ... are k and w dependent $\Delta(\epsilon) = -\int_{\epsilon'}^{\infty} \Delta(\epsilon') K(\epsilon,\epsilon') \tanh \beta \epsilon' d\epsilon'$ $K(\epsilon,\epsilon') = \frac{\Omega}{2(2\pi)^2} \frac{m}{\pi^2} \int_{q}^{q} \sqrt{(q,\omega)} dq$ [K] = [NV] = [N] - [K]REPULSIVE COULOND INTERACTION $K^{c} = [h] \sim \Re \left\{ q \cdot \frac{4\pi e^{2}}{\Re q^{2} \in (q, \omega)} dq \right\}$ LINDHARD E KC tEn - Er La U



BEYOND BOS MODEL (CONT'D) (FREQUENCY DEPENDENCE) ISOTOPE EFFECT BCS MODEL TENTE - TO AL Const TM TC ~ M-K Q = 1/2 BCS MODEL 2 SQ WELL MODEL TE ~ TO E - A-M+ $\alpha = \frac{1}{2} \left[1 - \left(\frac{M^*}{\lambda \cdot M^*} \right)^2 \right] \implies \alpha \leq \frac{1}{2}$ FOR M#= 1; q=0 WAVEVECTOR DEPENDENCE TOTAL DIELECTRIC FUNCTION $\in (q, \omega)$ can include electron-electron and electron-phonon effects e.g. $V_{RR'}^{T} = \frac{V_{q}}{\varepsilon_{q,w}} = \frac{4\pi e^{2}}{\Gamma g^{2}} \frac{1}{\varepsilon_{e} + \varepsilon_{p}}$ $\frac{1}{\epsilon_e + \epsilon_p} = \frac{1}{\epsilon_e} - \frac{\epsilon_p}{\epsilon_e(\epsilon_e + \epsilon_p)} \quad \begin{array}{c} & & \\ & &$









IMPORTANT TO KNOW (FROM EXPERIMENT) WHICH MODEL APPLIES exp: 20, tunneling, Cr(T)...



Superconductivity in diamond

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 ³Lebedev Physics Institute, Russian Academy of Sciences, 117924 Moscow, Russia

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ARTICLES

Quasiparticle dynamics in graphene

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Graphene Electronic Structure







Wannier Representation

 $g_{mn}^{\nu}(\mathbf{k},\mathbf{q}) = \langle m\mathbf{k} + \mathbf{q} | \Delta V_{\mathbf{q}\nu} | n\mathbf{k} \rangle$

$$\begin{split} |n\mathbf{k}\rangle &= \frac{1}{N_{\rm e}} \sum_{m\mathbf{R}_{\rm e}} e^{i\mathbf{k}\cdot\mathbf{R}_{\rm e}} U_{mn,\mathbf{k}}^{\dagger} |m\mathbf{R}_{\rm e}\rangle \\ \Delta_{\mathbf{q}\nu} V(\mathbf{r}) &= \sum_{\mathbf{R}_{\rm p}} \exp(i\mathbf{q}\cdot\mathbf{R}_{\rm p}) \sum_{\kappa\alpha} M_{\kappa}^{-\frac{1}{2}} e_{\mathbf{q}\nu}(\kappa\alpha) \ \Delta_{\kappa\alpha,\mathbf{R}_{\rm p}} V(\mathbf{r}) \end{split}$$

Wannier Representation

$$g(\mathbf{k}, \mathbf{q}) = \sum_{\mathbf{R}_{e}, \mathbf{R}_{p}} e^{i\mathbf{k}\cdot\mathbf{R}_{e}} e^{i\mathbf{q}\cdot\mathbf{R}_{p}} u_{\mathbf{q}} U_{\mathbf{k}+\mathbf{q}} g(\mathbf{R}_{e}, \mathbf{R}_{p}) U_{\mathbf{k}}^{\dagger}$$
Bloch
Wannier





JAN 28, 2007

I have proposed that we take steps to videotape the entire workshop. My colleagues are of 2 minds regarding this suggestion: one is that people will keep their best ideas to themselves; the other is that with a recording of our deliberations, there will be unambiguous proof of the genesis of any idea that leads us to the Promised Land. I clearly support the latter view, but I welcome your comments on this issue.

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2) IF WHAT I SAY BECOMES THE GENESIS OF ANY IDEA THAT LEADS THE " CADRE OF YOUNGER SCIENTISTS" TO THE PROMISED LAND, UNLIKE MOSES, I'M GOING IN WITH THEM!



"To be honest, I never would have invented the wheel if not for Urg's groundbreaking theoretical work with the circle."

World Year of Physics 2005

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www.physics2005.org

1. Proposed method for measurement of "c", using interferometer. Proposed to Prof. Dr. Weber (1900)

(Comment: This was later done by A. A. Michelson who did the experiment and became the first American to be awarded the Nobel Prize [1907])

 Proposed method for measurement of "c", using interferometer. Proposed to Prof. Dr. Weber (1900).
 REJECTED

- Proposed method for measurement of "c", using interferometer. Proposed to Prof. Dr. Weber.
 REJECTED
- 2. Proposed investigation of the effect of heat on electrical conductance of solids. Proposed to Prof. Dr. Weber.

(Comment: very important for electronics)

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- Proposed investigation of the effect of heat on electrical conductance of solids. Proposed to Prof. Dr. Weber.
 REJECTED
- 3. Performed theoretical work on the thermal conductivity of solids. Wrote up as dissertation. Submitted to Prof. Dr. Weber.

(Comment: again important for electronics and heat applications)

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 REJECTED
- Proposed investigation of the effect of heat on electrical conductance of solids. Proposed to Prof. Dr. Weber.
 REJECTED
- Performed theoretical work on the thermal conductivity of solids. Wrote up as dissertation. Submitted to Prof. Dr. Weber.
 REJECTED (written on wrong kind of paper?)

4. Performed theoretical work on extension of intermolecular forces from liquids to gases. Wrote up as dissertation. Presented to Prof. Dr. Kleiner.

(Comment: a fundamental theory of how matter behaves)

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 REJECTED

- Performed theoretical work on extension of intermolecular forces from liquids to gases. Wrote up as dissertation. Presented to Prof. Dr. Kleiner.
 REJECTED
- 5. Proposed investigation on electrodynamics of moving bodies. Proposed to Prof. Dr. Kleiner.

(Comment: this later led to Einstein's famous equation)



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- Proposed investigation on electrodynamics of moving bodies. Proposed to Prof. Dr. Kleiner.
 REJECTED
- 6. Performed theoretical work on measuring the molecular dimensions of sugar molecules. Wrote up as dissertation. Submitted to Profs. Kleiner and Burkhardt.

(Comment: important to prove the existence of atoms and molecules)

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REJECTED (too short, 17 pages)

 Added one sentence and resubmitted theoretical work on measuring the molecular dimensions of sugar molecules. Submitted to Profs. Kleiner and Burkhardt. Added one sentence and resubmitted theoretical work on measuring the molecular dimensions of sugar molecules. Submitted to Profs. Kleiner and Burkhardt.

ACCEPTED (1905) !

ALBERT EINSTEIN

"The most valuable tool of the theoretical physicist is his wastebasket."

THE END