Exciton Mechanism of Superconductivity

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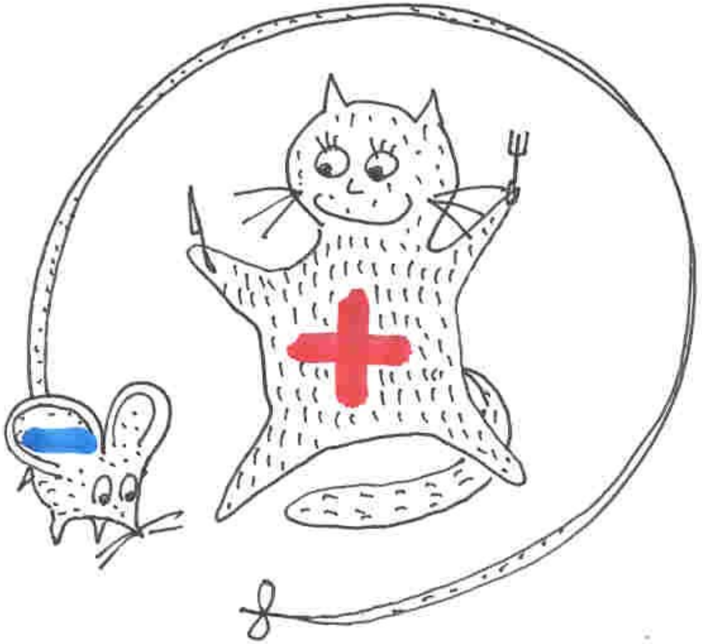
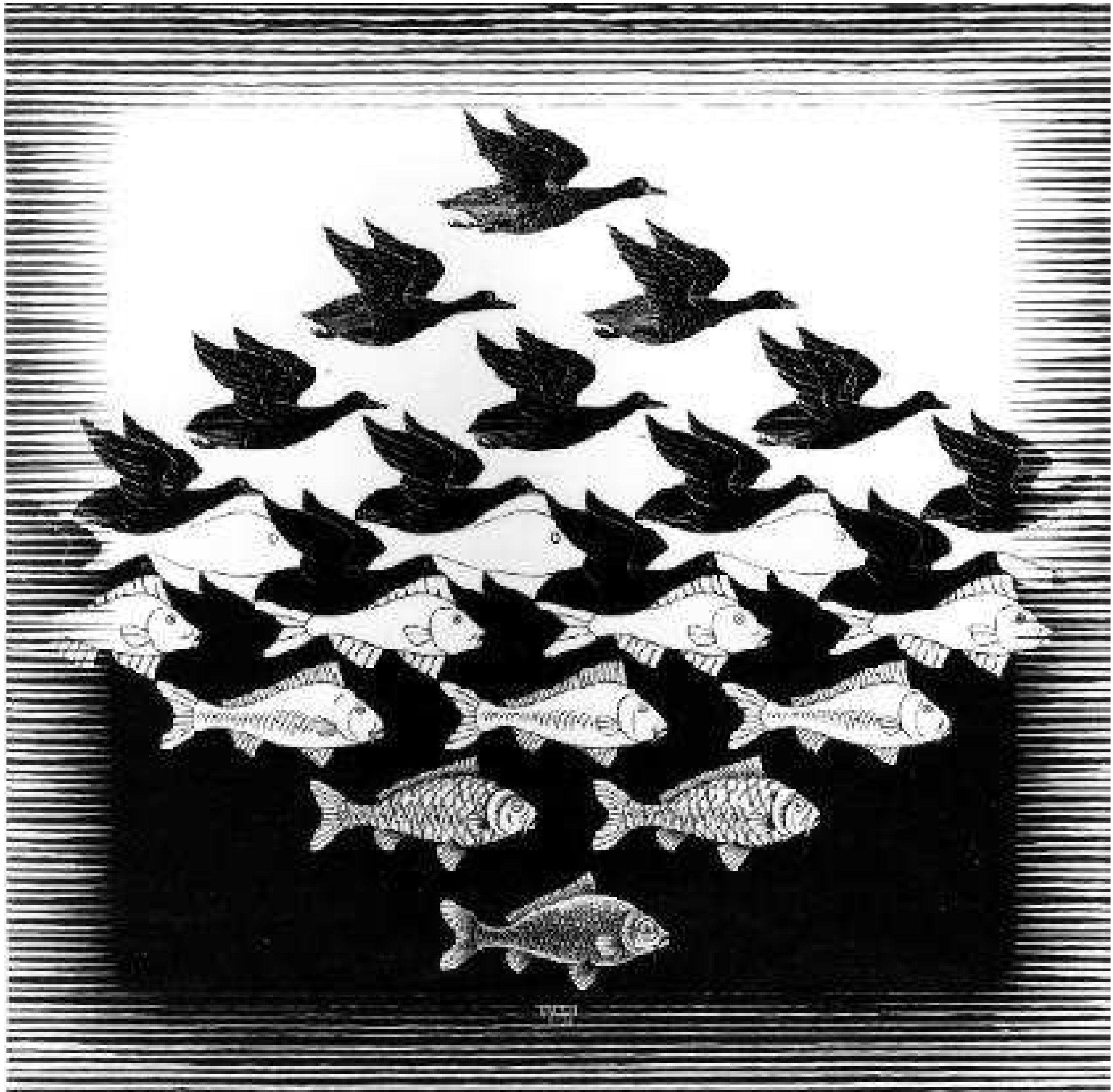
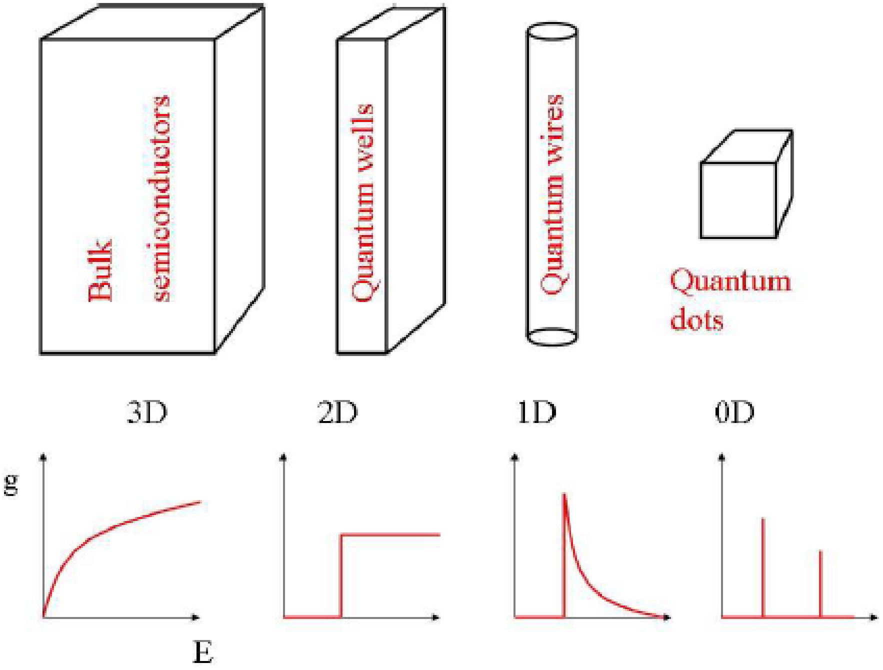




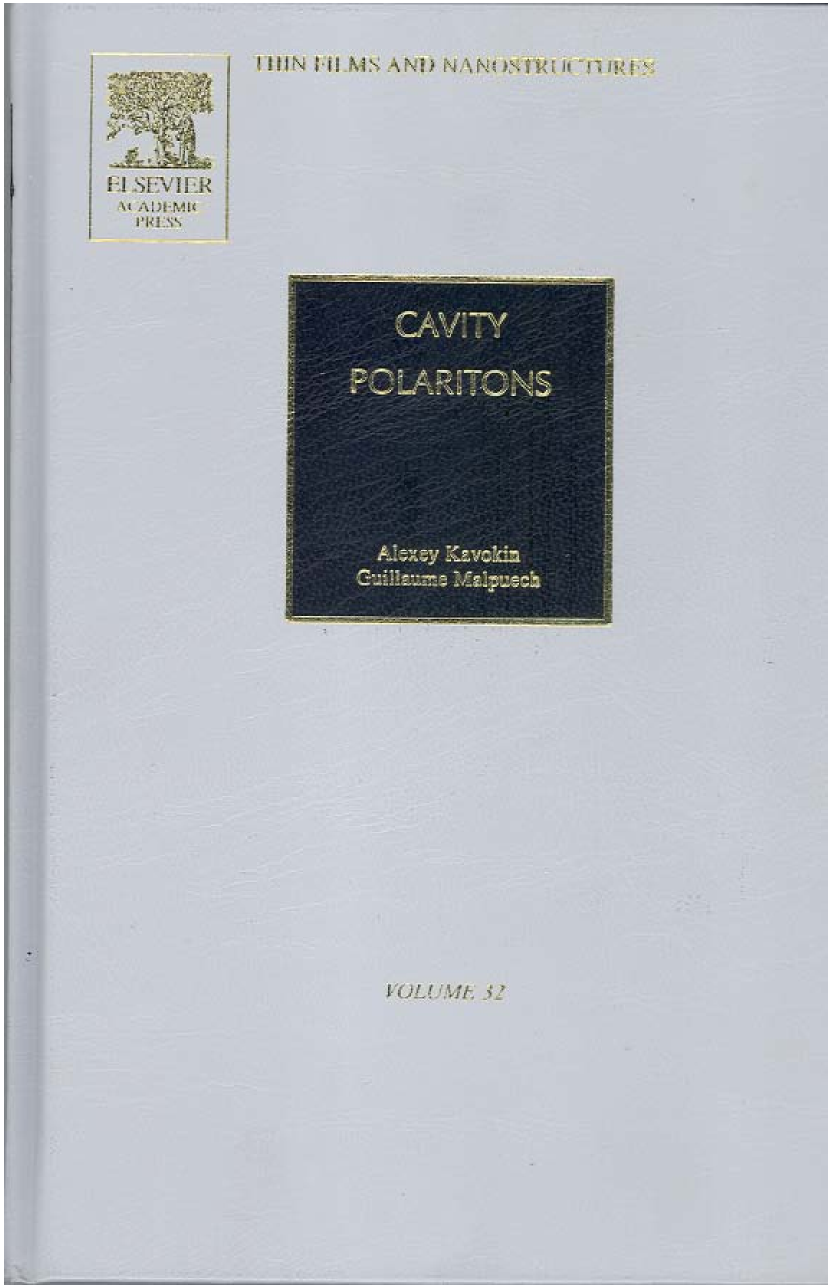
My background:

Excitons in confined structures



My PhD supervisor Prof. E.L. Ivchenko

E.L.Ivchenko, A.V.Kavokin, Light Reflection from Quantum Well, Quantum Wire and Quantum Dot Structures, *Sov.Phys.Solid State* **34**, 1815-1822, (1992) Since 1992 I worked in semiconductor optics…



THIN FILMS AND NANOSTRUCTURES

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POLARITONS

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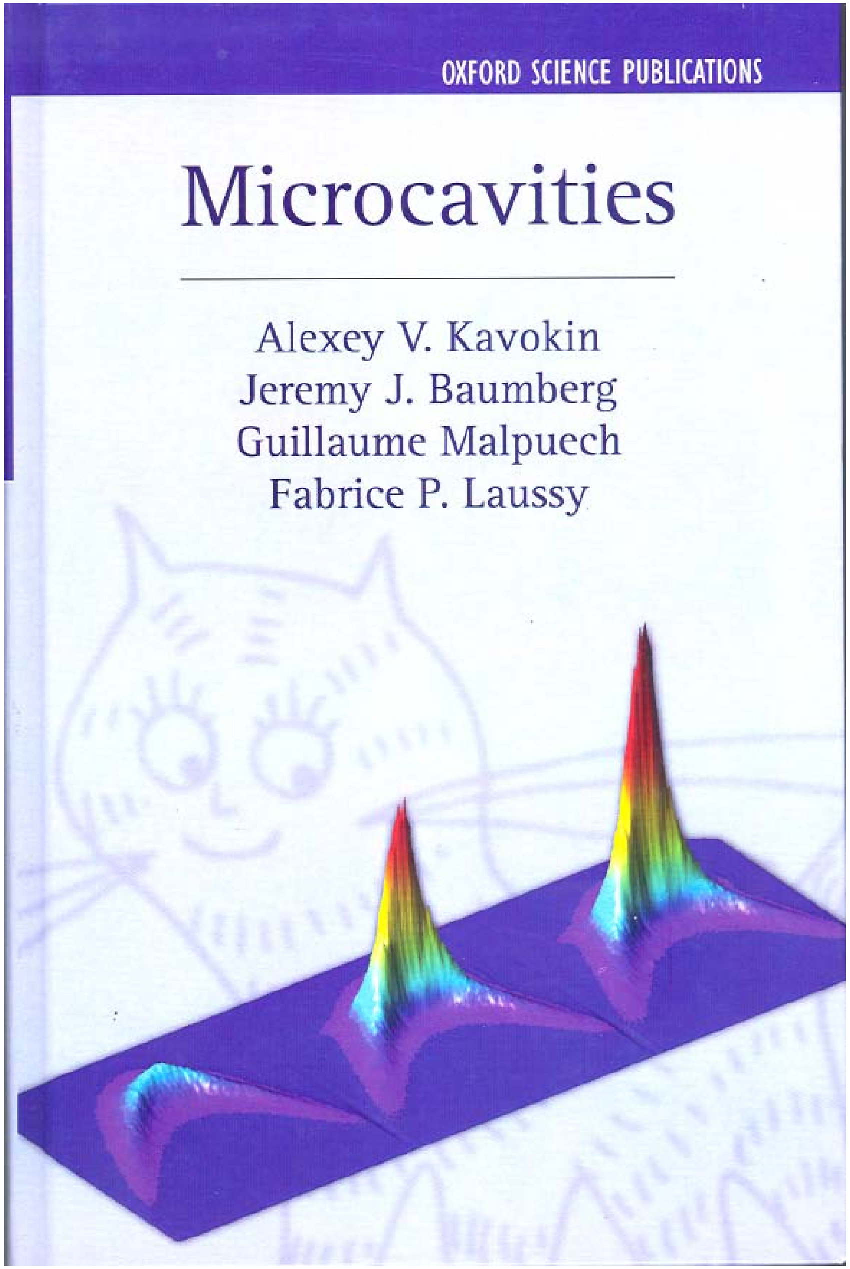
VOLUME 32

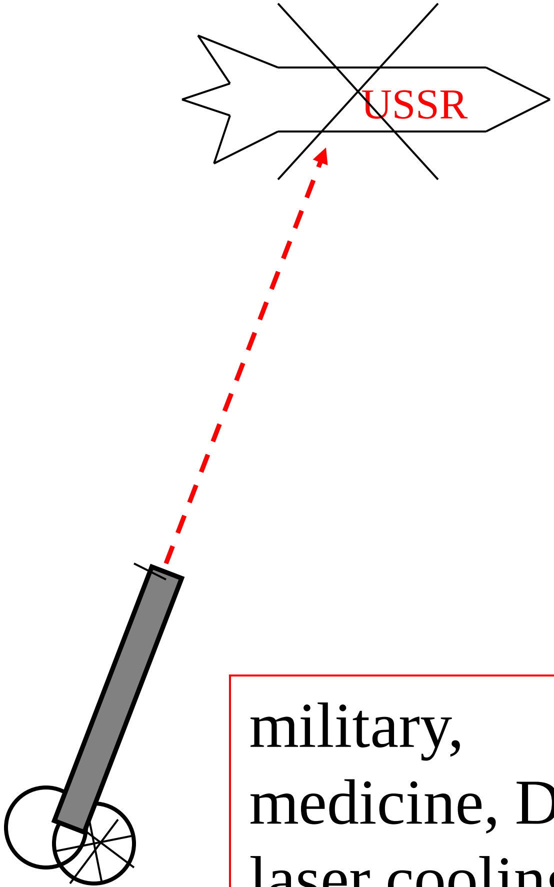


OXFORD SCIENCE PUBLICATIONS

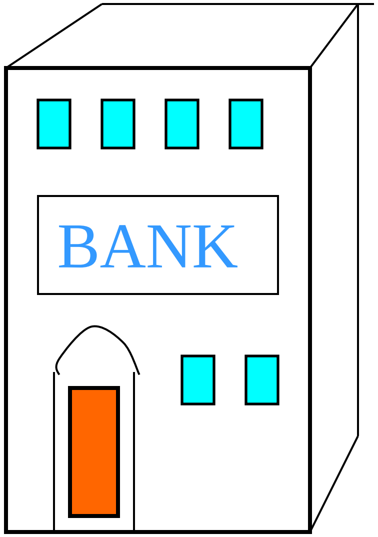
Microcavities

Alexey V. Kavokin
Jeremy J. Baumberg
Guillaume Malpuech
Fabrice P. Laussy

High power lasers and low power lasers



APPLICATIONS



military,
medicine, DVD,
laser cooling...

Single photon sources,
quantum cryptography,
quantum computing...

I have chosen low power lasers... And this brought me to superconductivity!

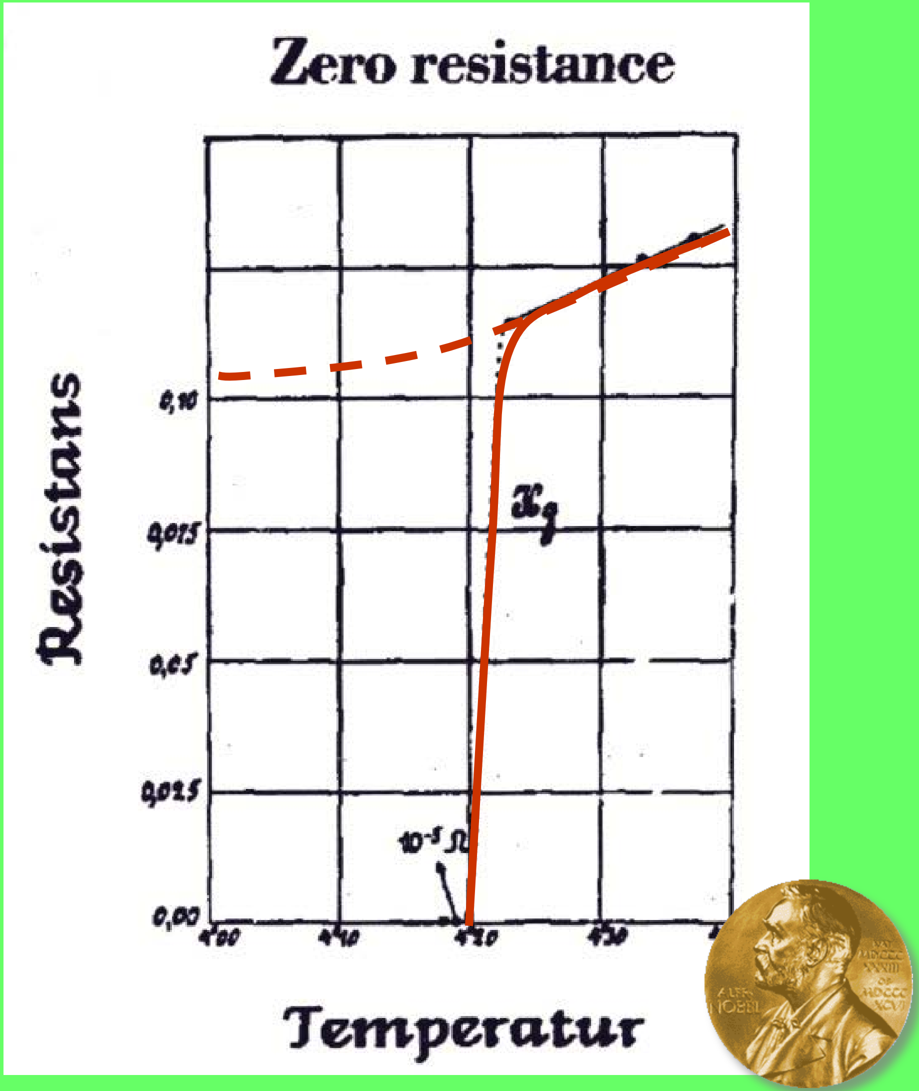
Syllabus of the course

- Bardeen-Cooper-Schriefer model
- Critical temperature for conventional superconductivity: ways to make it higher
- Band structure of semiconductors
- Wannier-Mott and Frenkel excitons
- Can excitons replace phonons in the BCS model?
- Retardation effect, problem of Coulomb repulsion
- Research on the Bose-Einstein condensation of excitons
- Exciton-polaritons in microcavities
- Hybrid metal-semiconductor structures
- Superconductivity mediated by a Bose-Einstein condensate of excitons
- Exciton-electron interaction potential
- Bardeen-Pines model
- Electron-electron attraction and repulsion
- Gap equation
- Bogolyubov approach to the solution of gap equation
- Predictions of critical temperature
- Strong coupling regime: quatrons
- Perspectives for experimental observation of the exciton-mediated superconductivity

Part 1 (overview of conventional superconductivity)

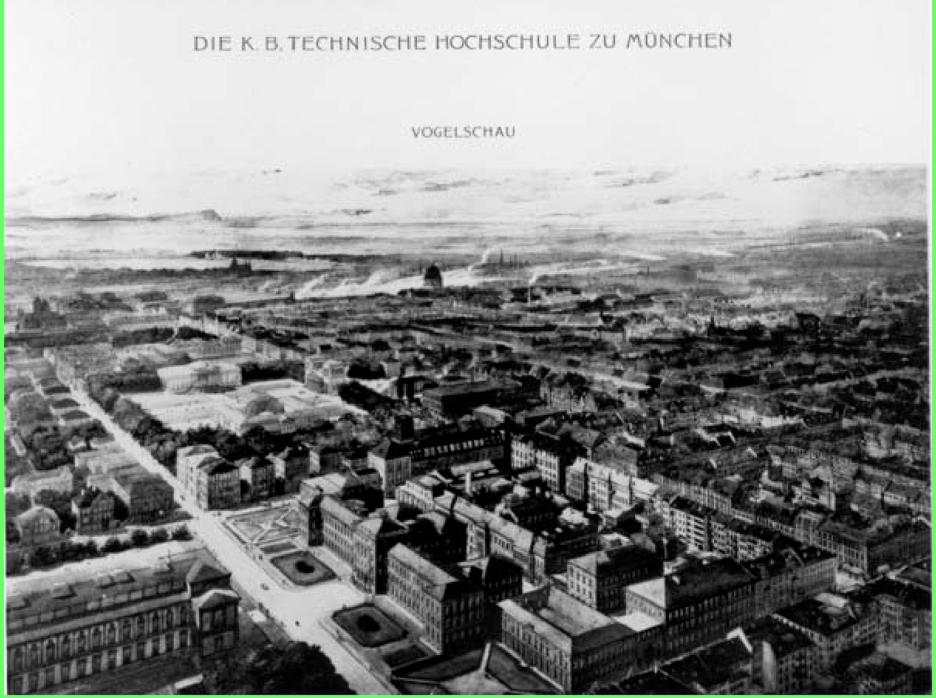
1911: discovery of superconductivity

- Discovered by **Kamerlingh Onnes** in 1911 during first low temperature measurements to liquefy helium
- Whilst measuring the resistivity of “pure” Hg he noticed that the electrical resistance dropped to zero at 4.2K
- In 1912 he found that the resistive state is restored in a magnetic field or at high transport currents



1913

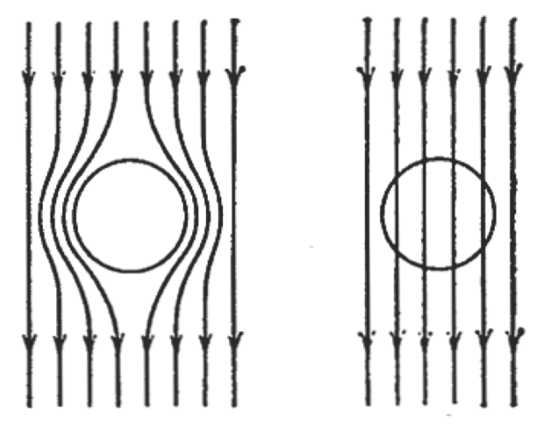
1933: Meissner-Ochsenfeld effect

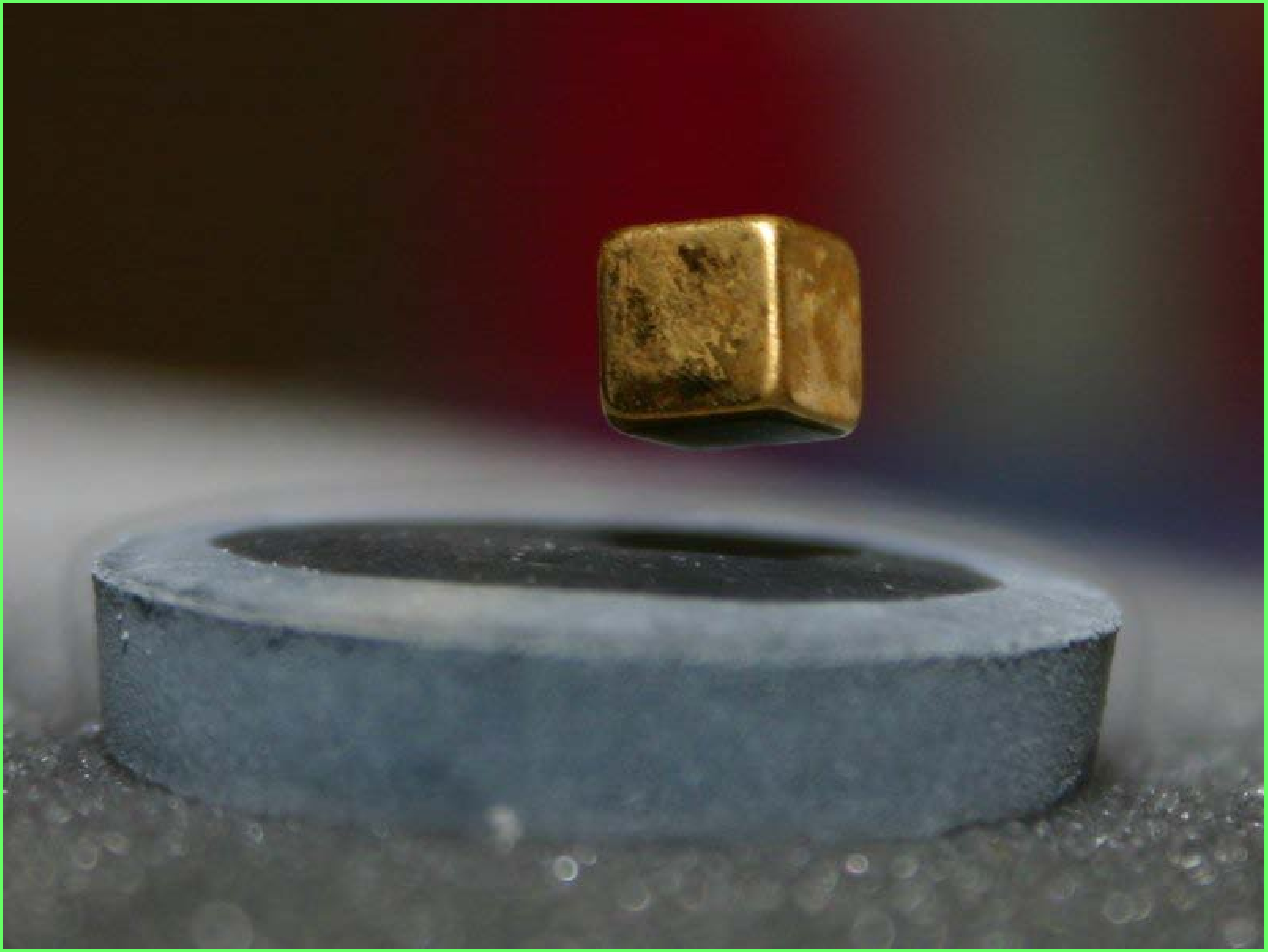


- The *Meissner-Ochsenfeld effect* (1933)

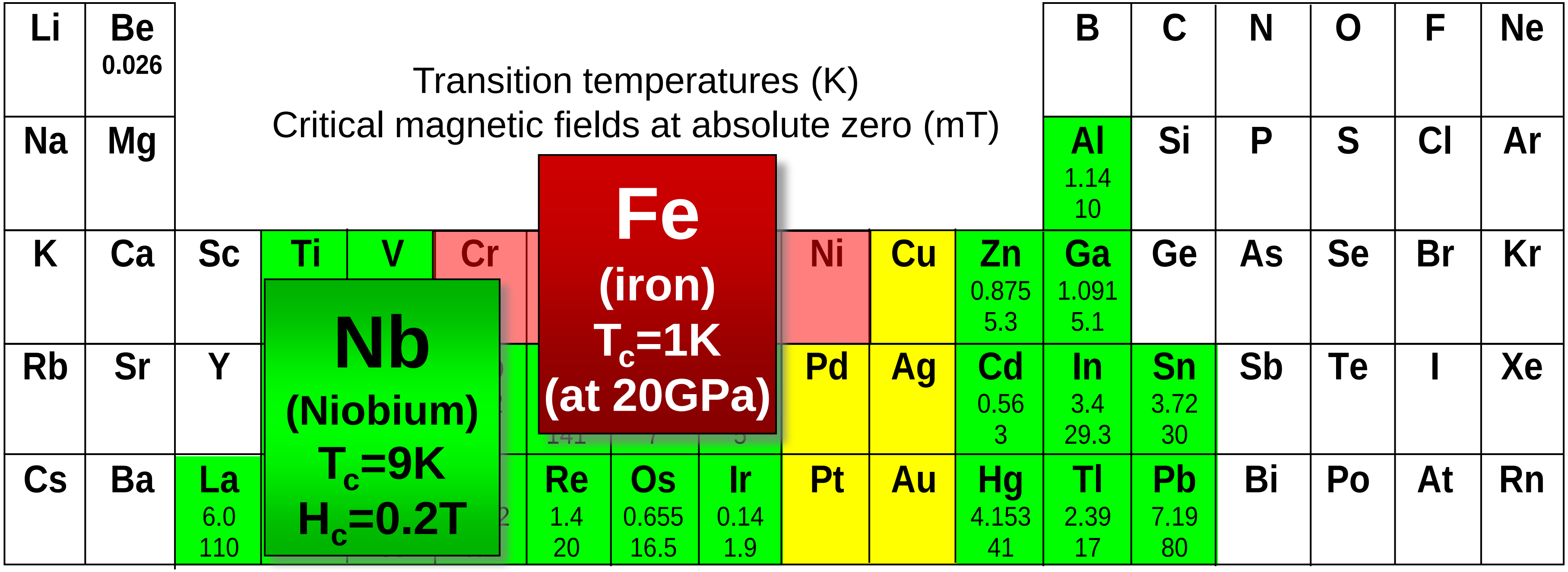
Magnetic field does not penetrate the sample

Ideal conductor! Ideal diamagnetic!





The superconducting elements



Transition temperatures (K)
Critical magnetic fields at absolute zero (mT)

Li	**Be** 0.026											**B**	**C**	**N**	**O**	**F**	**Ne**
Na	**Mg**											**Al** 1.14 10	**Si**	**P**	**S**	**Cl**	**Ar**
K	**Ca**	**Sc**	**Ti**	**V**	**Cr**				**Ni**	**Cu**	**Zn** 0.875 5.3	**Ga** 1.091 5.1	**Ge**	**As**	**Se**	**Br**	**Kr**
Rb	**Sr**	**Y**				141	7	5	**Pd**	**Ag**	**Cd** 0.56 3	**In** 3.4 29.3	**Sn** 3.72 30	**Sb**	**Te**	**I**	**Xe**
Cs	**Ba**	**La** 6.0 110				**Re** 1.4 20	**Os** 0.655 16.5	**Ir** 0.14 1.9	**Pt**	**Au**	**Hg** 4.153 41	**Tl** 2.39 17	**Pb** 7.19 80	**Bi**	**Po**	**At**	**Rn**

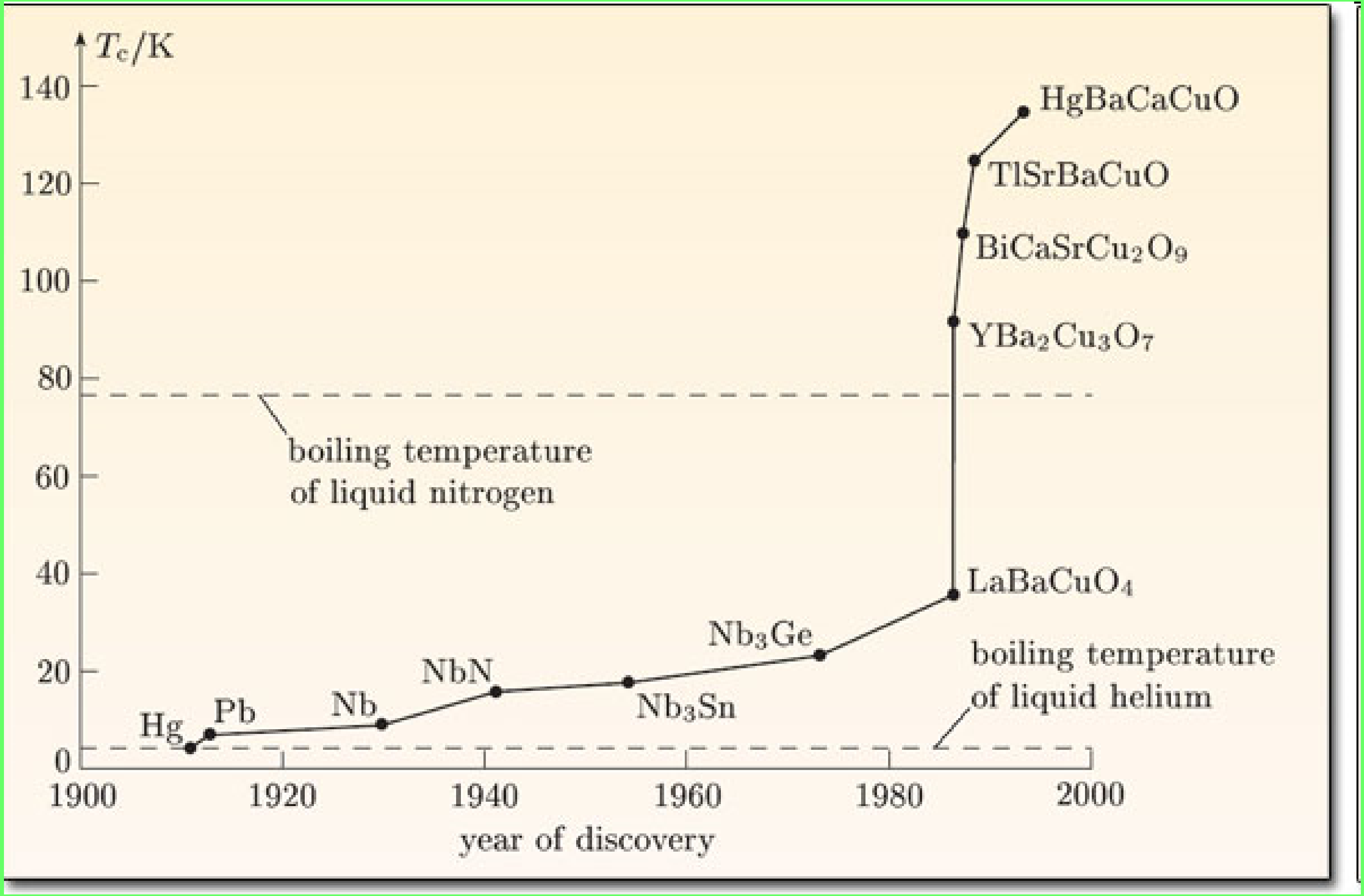
Nb
(Niobium)
T_c=9K
H_c=0.2T

Fe
(iron)
T_c=1K
(at 20GPa)

- Transition temperatures (K) and critical fields are generally low
- Metals with the highest conductivities are not superconductors
- The magnetic 3d elements are not superconducting

...or so we thought until 2001

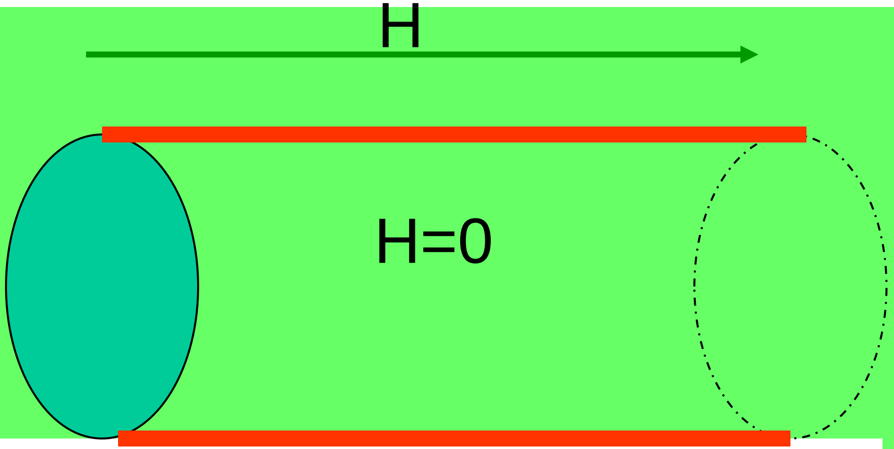
Superconductivity in alloys



1935: Brothers London theory

$$\partial(\Lambda \boldsymbol{j})/\partial t = \boldsymbol{E}.$$

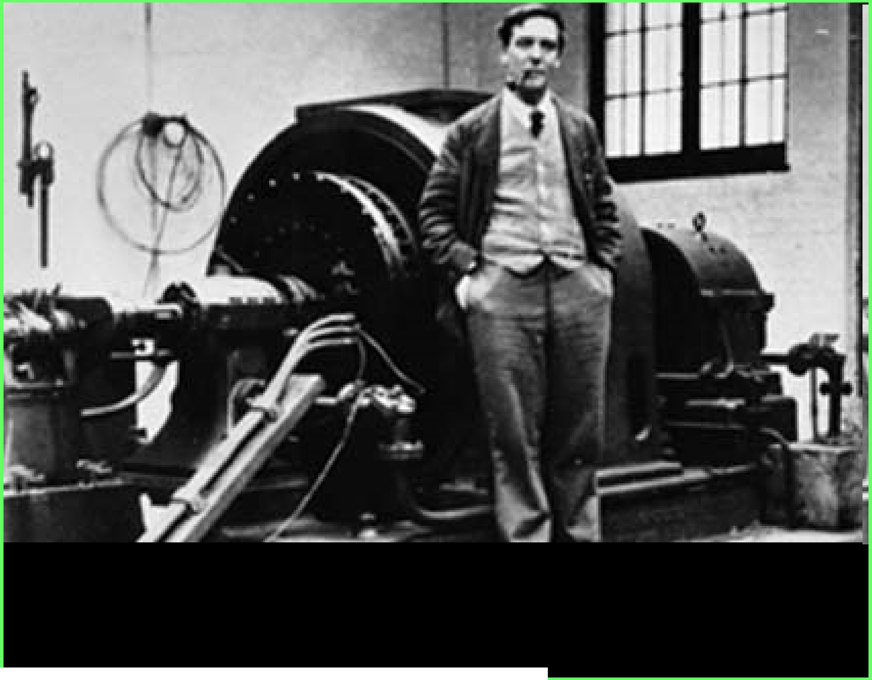
$$(\partial/\partial t)(\mathrm{rot}\,\Lambda \boldsymbol{j} + c^{-1}\boldsymbol{H}) = 0.$$



$$H = H_0 \exp(-x/\delta),$$

$$j_y = (cH_0/4\pi\delta)\exp(-x/\delta).$$

1937: Superfluidity of liquid He_4



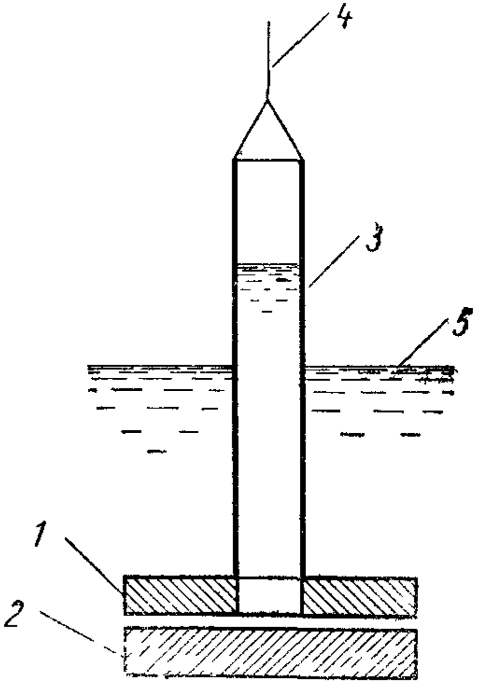


Рис. 5.6. Схема опыта Капицы по измерению вязкости жидкого гелия [Доклады АН СССР, **1938**, т. XVIII, № 1, с. 22].

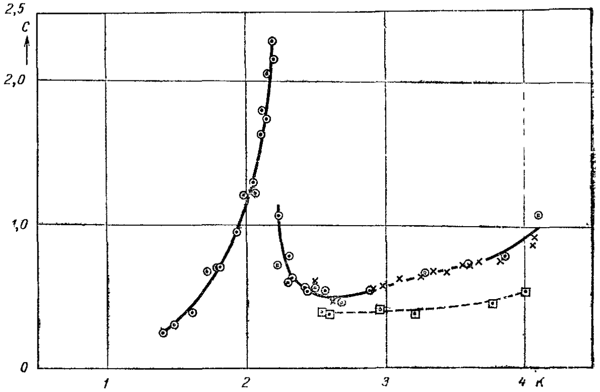


Рис. 5.3. Зависимость теплоемкости жидкого гелия (кал/г · K) от температуры.

Представлены результаты, полученные Кеезомом и Клузиусом; квадратики — теплоемкость при постоянном объеме, кружки — теплоемкость при постоянном давлении насыщенных паров гелия. Крестиками отмечены результаты ранних измерений теплоемкости при давлении насыщенных паров гелия, выполненных Даном и Камерлинг-Оннесом [*Commun. Phys. Lab. Univ. Leiden*, № 219e (1932), стр. 51, рис. 3].

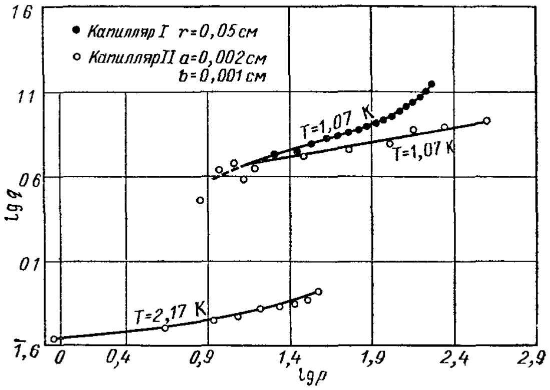
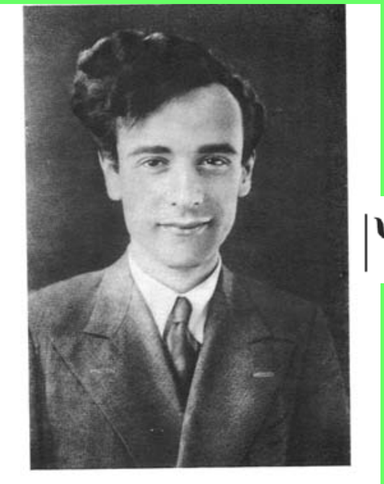


Рис 5 7 Зависимость (в логарифмическом масштабе) скорости (см/с) от давления (дина/см^2) при течении жидкого гелия II в капиллярах [*Nature*, **141** (1938), стр. 75, рис без номера].

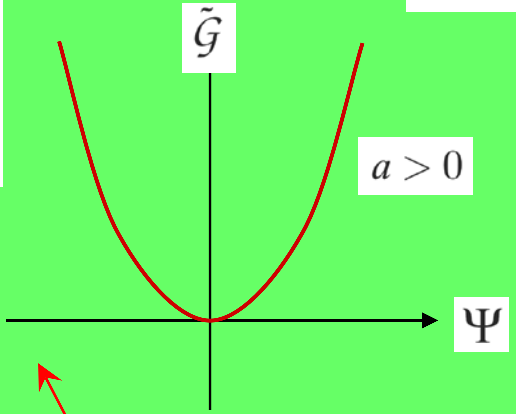
1978

Landau theory of 2nd order phase transitions

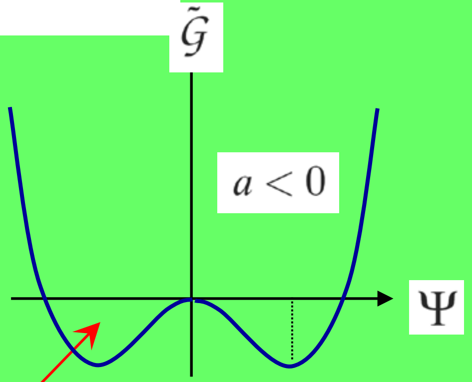


Order parameter? Hint: wave function of Bose condensate (complex!)

$$|\Psi|^2 \ll 1 \Rightarrow \mathcal{G}_s = \mathcal{G}_n + a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \cdots .$$



$$\tilde{\mathcal{G}} \equiv \mathcal{G}_s - \mathcal{G}_n$$



1962

a should change the sign at the transition point

Introduce $\tau = \frac{T-T_c}{T_c}$. Near T_c, $|\tau| \ll 1$: $a = \alpha\tau\,, \quad \alpha > 0.$

$$\begin{array}{lll} \Psi = 0 & \text{at} & T > T_c, \\ |\Psi|^2 = -(\alpha/b)\tau = |\Psi_0|^2 & \text{at} & T < T_c \end{array} .$$

1950: Ginzburg-Landau Phenomenology Ψ-Theory of Superconductivity

Order parameter? Hint: wave function of Bose condensate (complex!)

2003

Inserting $|\Psi_0|^2$ and using the energy conservation law

$$\mathcal{G}_s - \mathcal{G}_n = \frac{(\alpha\tau)^2}{2b} = \frac{H_c^2}{8\pi}.$$

How one can describe an inhomogeneous state?

One could think about adding $|\nabla\Psi|^2$. However, electrons are charged, and one has to add a gauge-invariant combination

$$\left|-i\hbar\nabla + \frac{2e}{c}\vec{A}\right|^2 \text{ where } \vec{H} = \text{curl } \vec{A}$$

Thus the Gibbs free energy acquires the form

$$\delta \mathcal{G} = \int dV \left\{ \alpha\tau|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{1}{4m}\left|\left(-i\hbar\nabla + \frac{2e}{c}\mathbf{A}\right)\Psi\right|^2 + \frac{H^2}{8\pi} \right\}$$

Ginzburg-Landau functional

To find distributions of the order parameter Ψ and vector–potential *A* one has to minimize this functional with respect to these quantities, i. e. calculate variational derivatives and equate them to 0.

Minimizing with respect to Ψ^*

$$(1/4m)\left[-i\hbar\nabla+(2e/c)\mathbf{A}\right]^2\Psi+\alpha\tau\Psi+b|\Psi|^2\Psi=0.$$

Minimizing with respect to A:

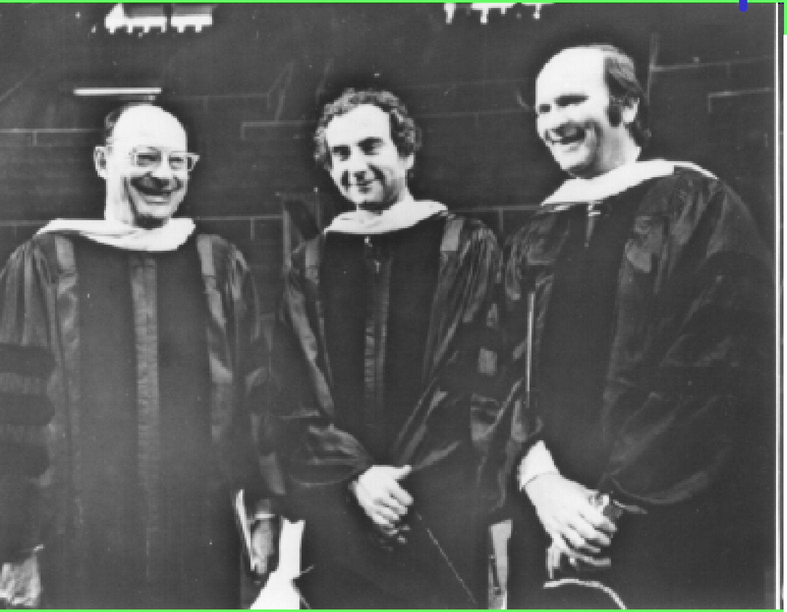
$$\text{curl curl }\mathbf{A}=\text{curl }\mathbf{H}=\frac{4\pi}{c}\mathbf{j}$$

Maxwell equation

$$\mathbf{j}=(ie\hbar/2m)\left(\Psi^*\nabla\Psi-\Psi\nabla\Psi^*\right)-(2e^2/mc)|\Psi|^2\mathbf{A}$$

The expression for the current indicates that the order parameter has a physical meaning of the wave function of the superconducting condensate.

1957: BCS- Microscopic theory of superconductivity



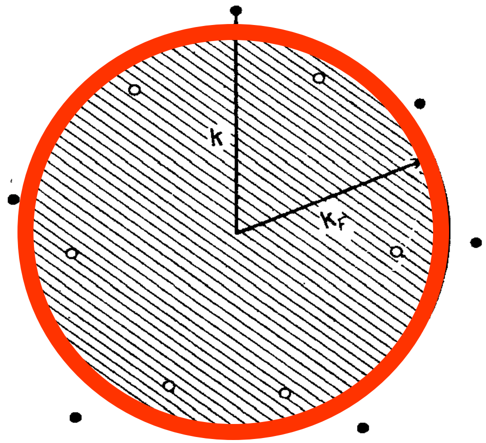
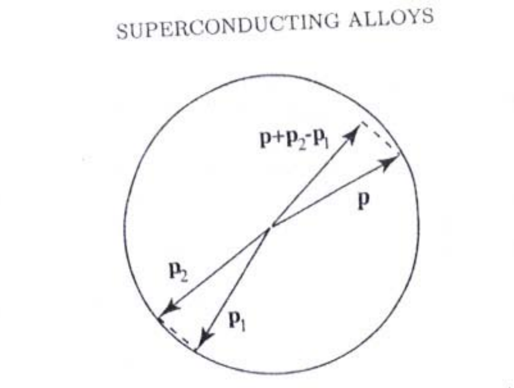


Рис. 3. Типичная возбужденная конфигурация нормального состояния.

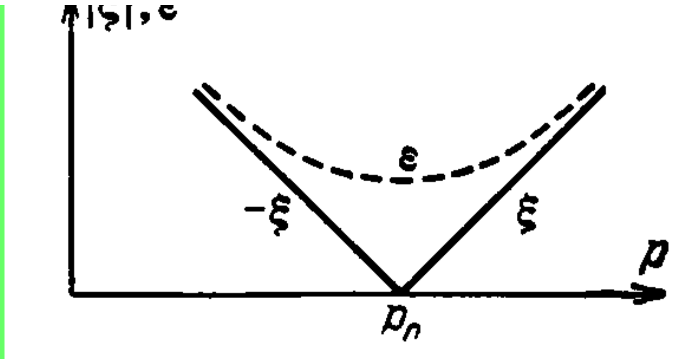
Квазичастичные возбуждения — заполненные состояния над поверхностью Ферми и дырки под поверхносью Ферми.



1972

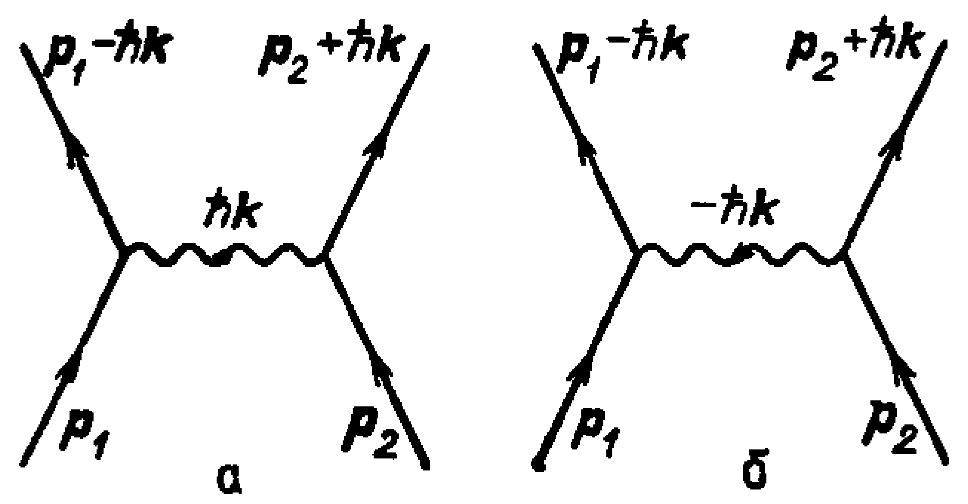
$$\langle V_{\mathrm{ph}} + V_{\mathrm{coul}} \rangle < 0,$$

$$T_c = (2\hbar\omega_D\gamma/\pi)\exp[-2/(gv)].$$

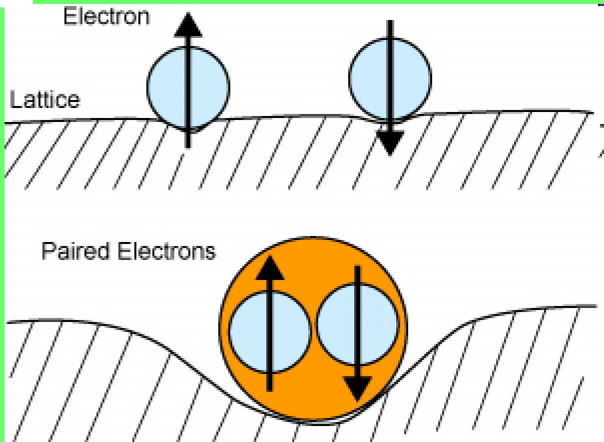


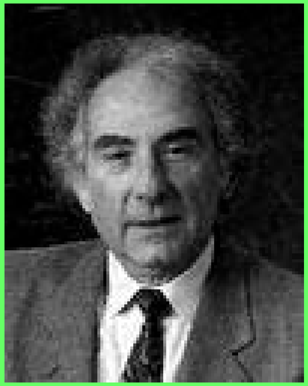
$$\Delta = \hbar\omega_D \exp[-2/(gv(\mu))].$$

1950:Electron phonon attraction

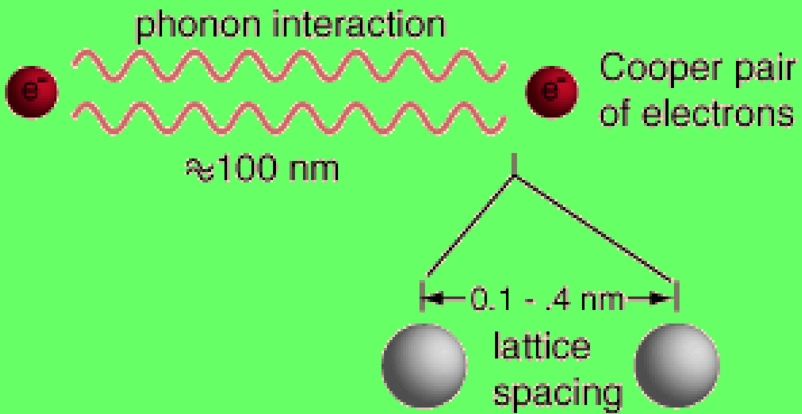


$$-\frac{\hbar^3}{p_0 mV}\,\frac{[\hbar\omega(k)]^2}{[\hbar\omega(k)]^2-[\varepsilon(p_1)-\varepsilon(p_1-\hbar k)]^2}\,.$$



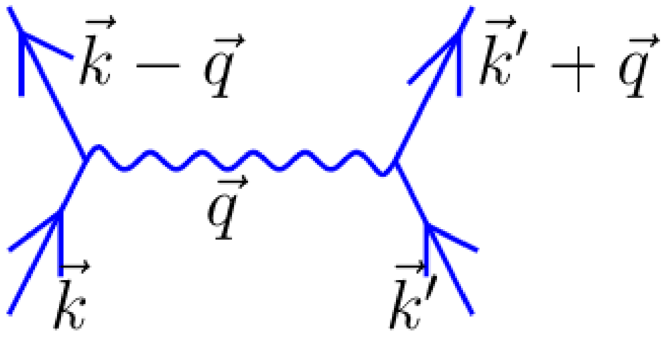


Cooper pairing in metals



Frölich Hamiltonian

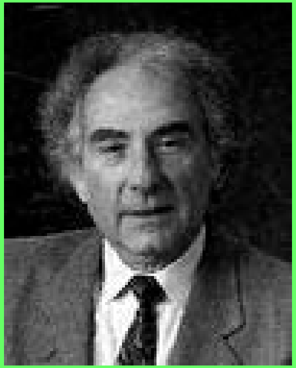
$$\sum_{\mathbf{k},\mathbf{q},\alpha} M(\mathbf{q})\sigma^{\dagger}_{\mathbf{k},\alpha}\sigma_{\mathbf{k}+\mathbf{q},\alpha}(b^{\dagger}_{-\mathbf{q}}+b_{\mathbf{q}})$$

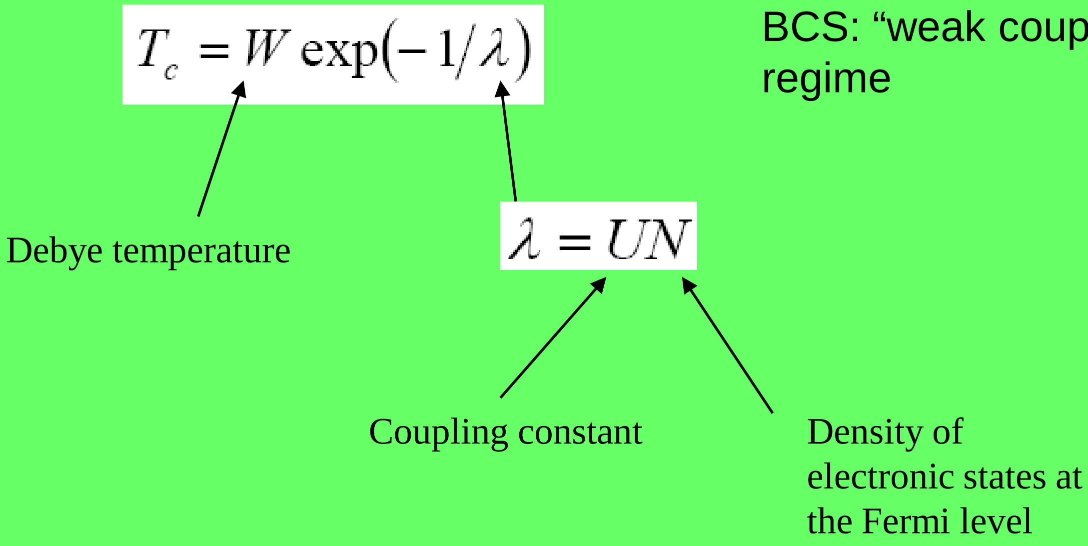


BCS model:

retarded interaction

Bardeen-Cooper-Schrieffer (BCS): Critical temperature:





$$T_c = W\exp(-1/\lambda)$$

Debye temperature

$$\lambda = UN$$

Coupling constant

Density of
electronic states at
the Fermi level

BCS: “weak coupling” regime

Debye temperatures:

Aluminium 428 K

Cadmium 209 K

Chromium 630 K

Copper 343.5 K

Gold 165 K

Iron 470 K

Lead 105 K

Manganese 410 K

Nickel 450 K

Platinum 240 K

Silicon 645 K

Silver 225 K

Tantalum 240 K

Tin (white) 200 K

Titanium 420 K

Tungsten 400 K

Zinc 327 K

Carbon 2230 K

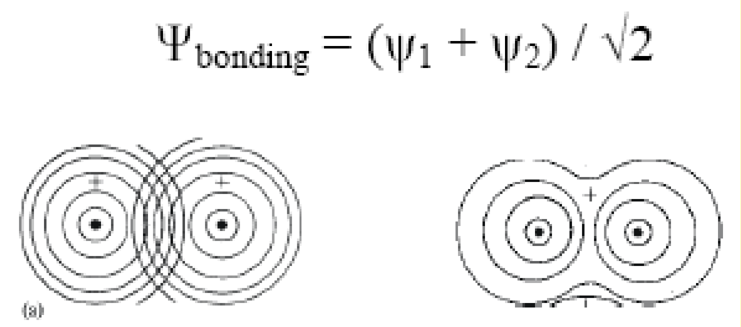
Ice 192 K

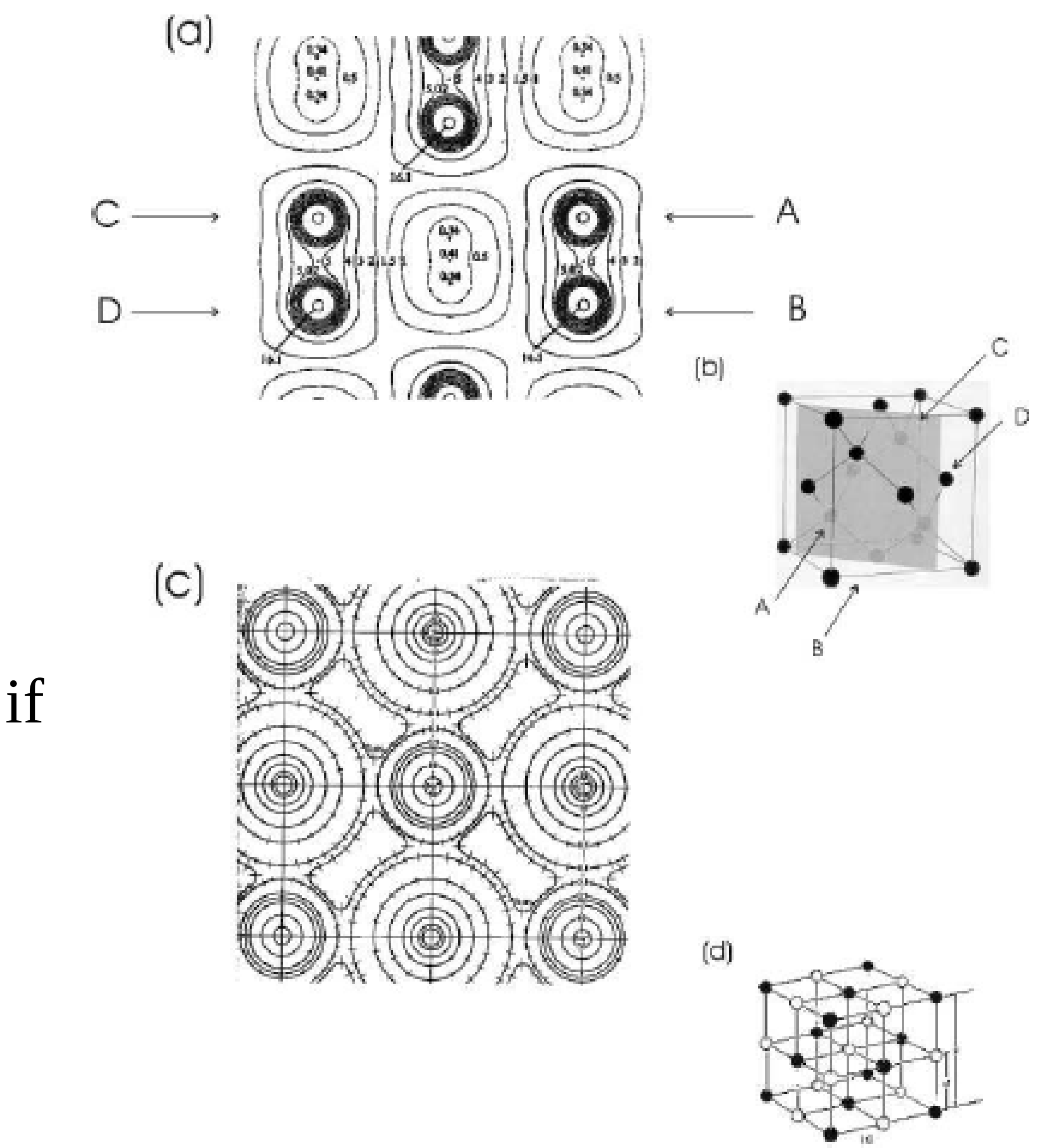
$$\lambda << 1$$

in conventional superconductors,

which is why the critical temperature is very low!

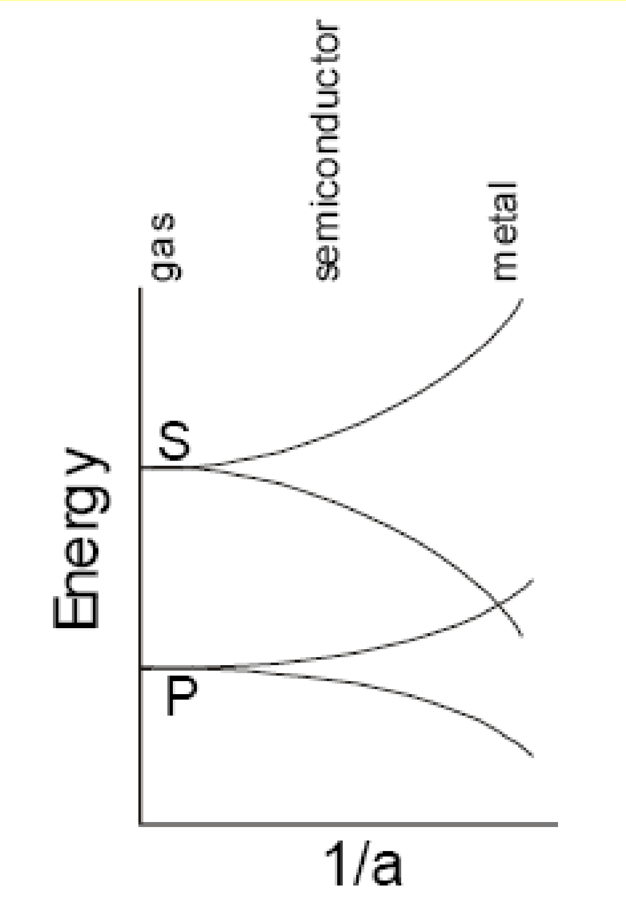
Part 2 Semiconductors

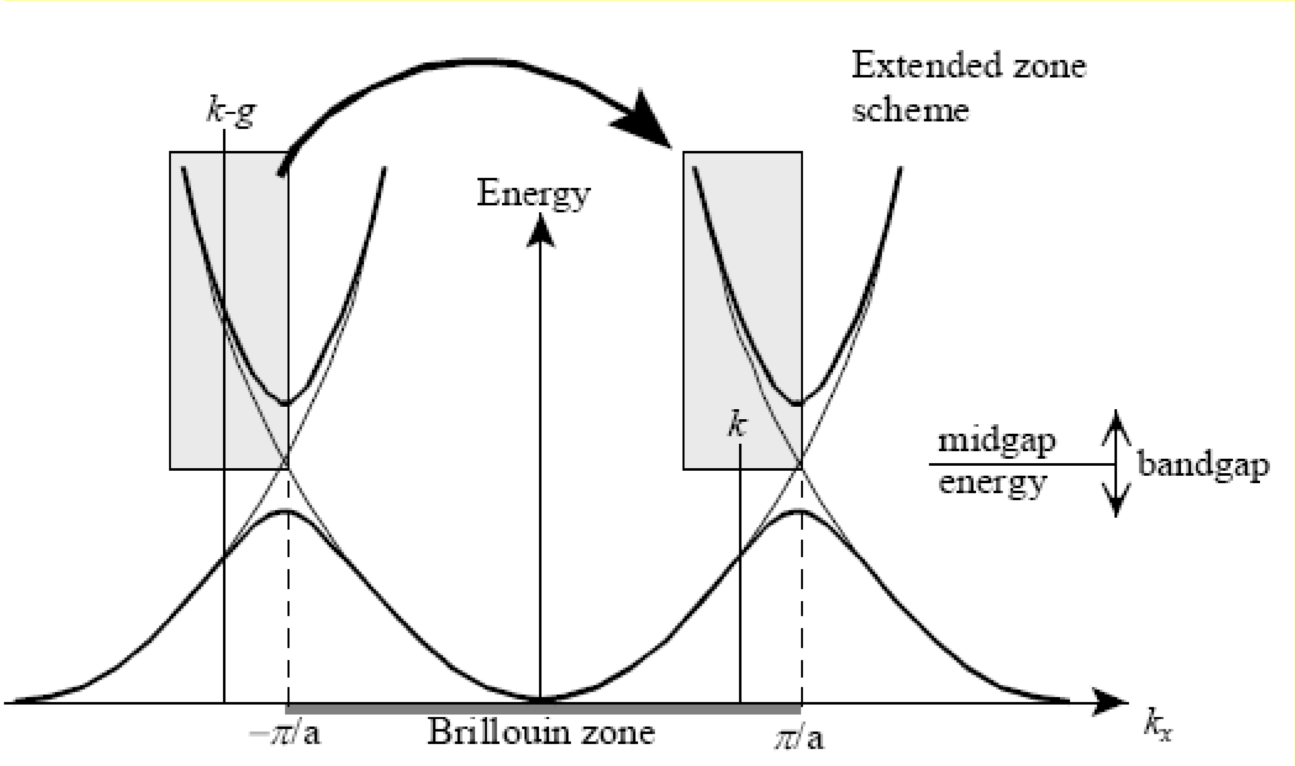




What happens with electrons if we put atoms close to each other?

Nearly free electrons: crystal bands. Tight binding model: crystal bands

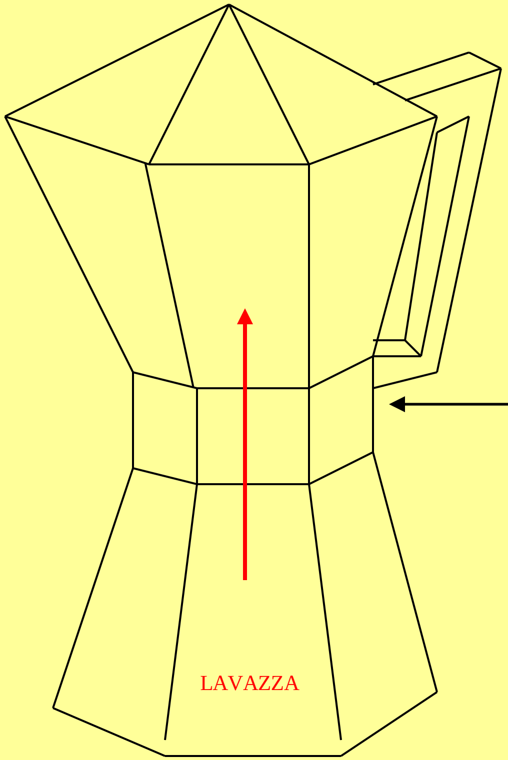




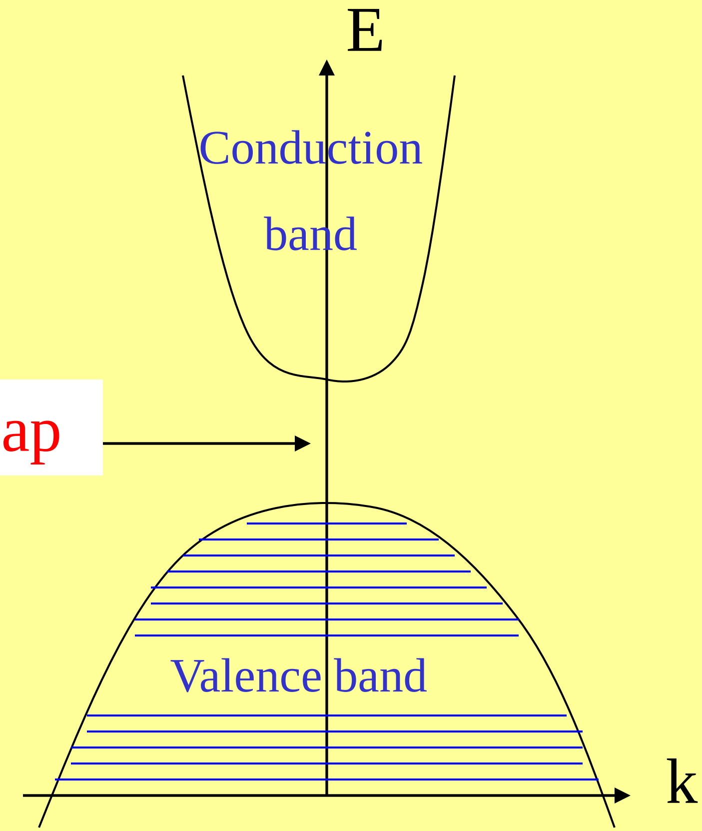
Band structure: metals, semiconductors, dielectrics

Italian coffee machine

Semiconductor band structure



Energy gap



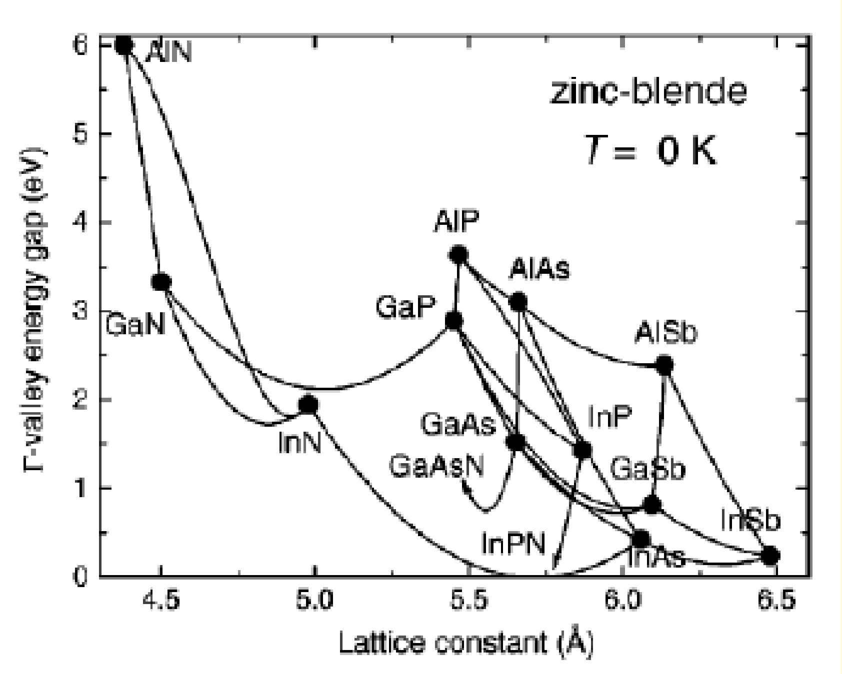
What happens if we heat them?

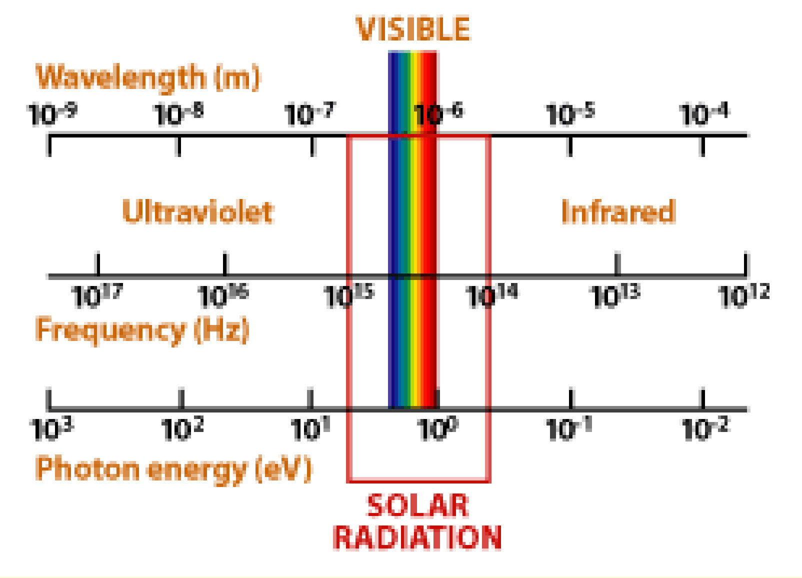
Quasiparticles in crystals

electrons →

holes →

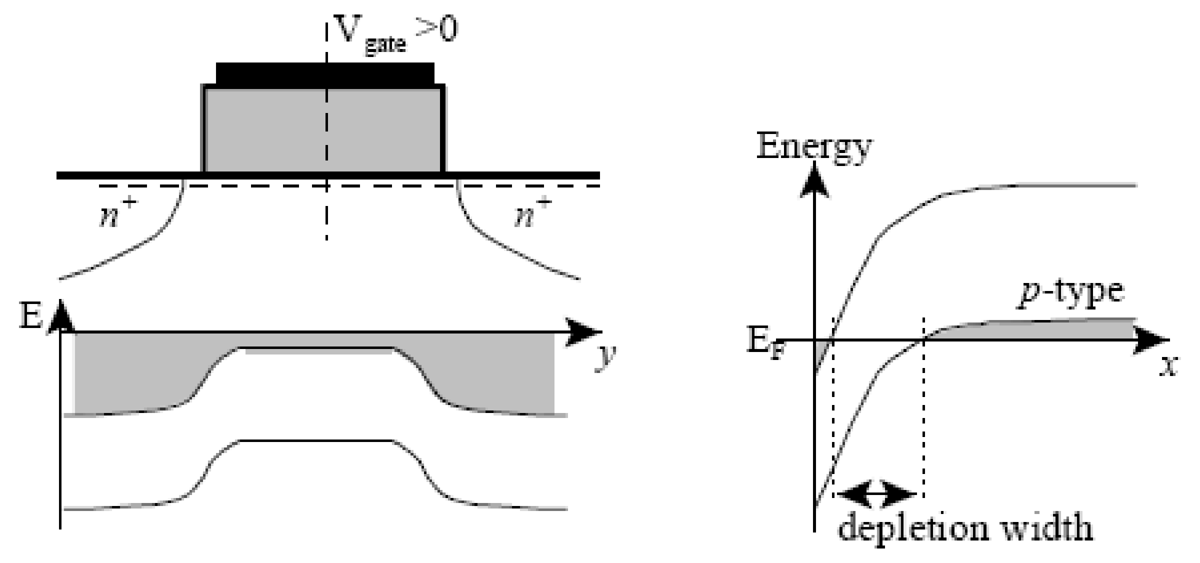
5. Semiconductors





because their bandgaps may be within the visible light spectrum !

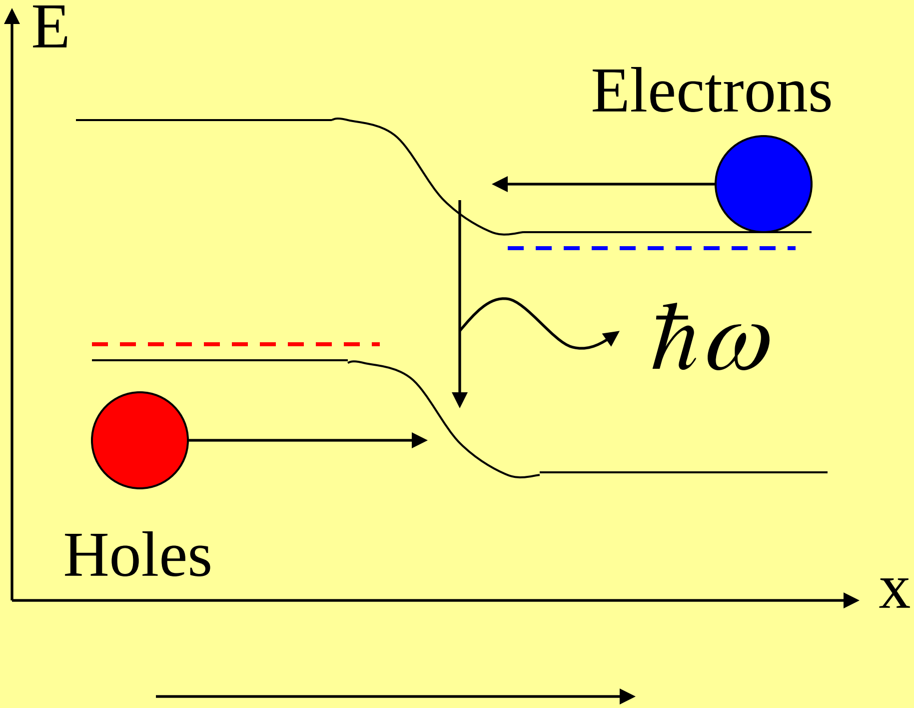
A transistor



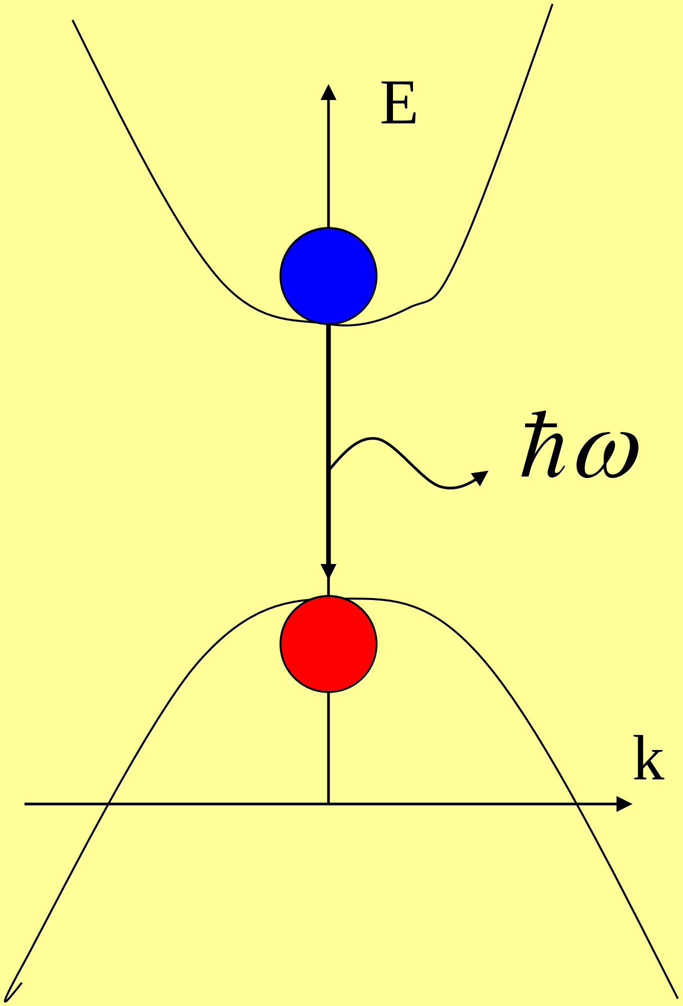
All modern electronics is based on semiconductor devices!

Semiconductor lasers

p-n junction

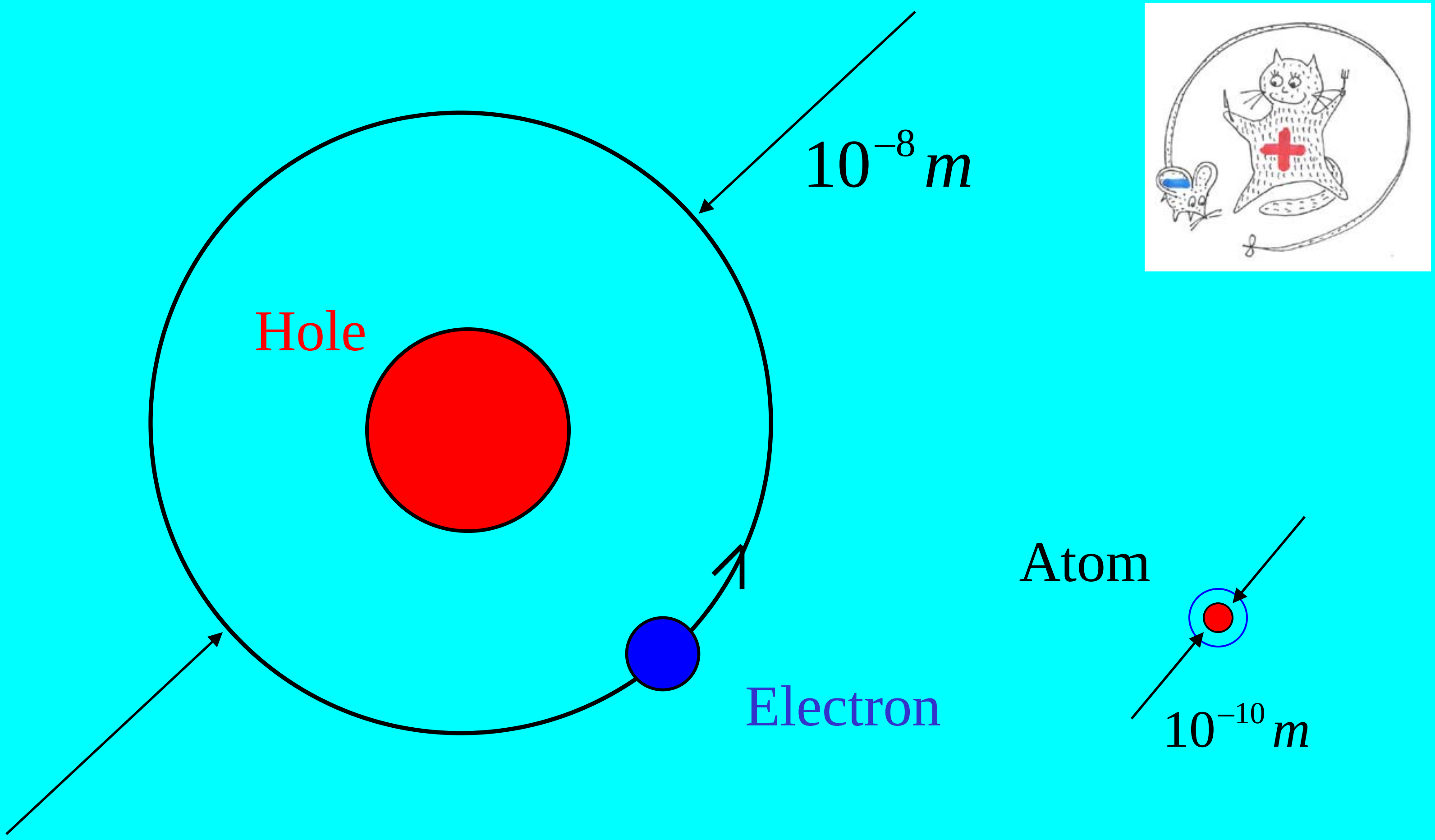


Electric field

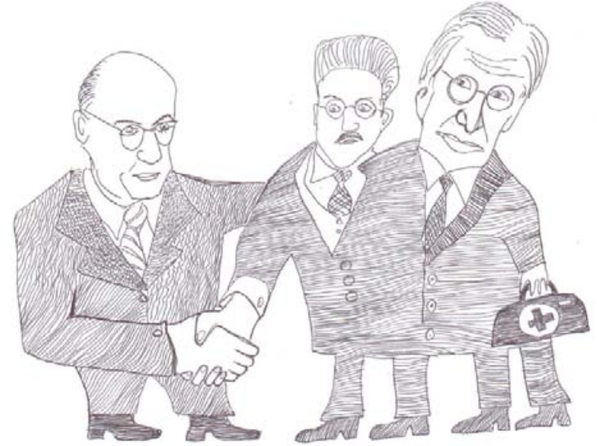


Part 3 Excitons in semiconductors

EXCITON: an artificial positronium ATOM made from an electron and a hole



Theoretical concept of excitons

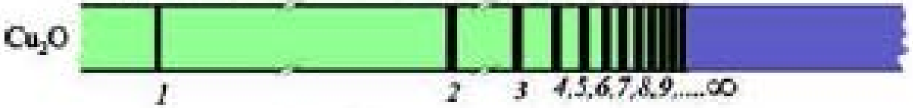


Yakov Il'ich **Frenkel** (1894–1952), Sir Nevill. Francis **Mott** (1905–1996) and Grégory **Wannier** (1911–1983) gave their name to the two main categories of excitons.

Organic molecular crystals

Inorganic semiconductor crystals

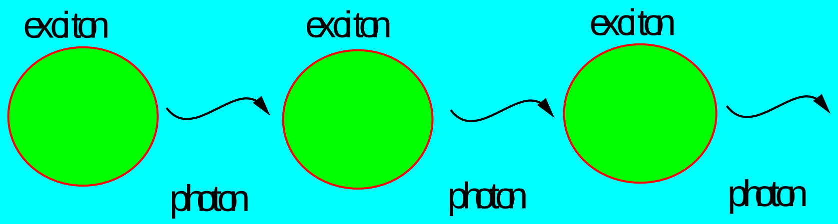
Discovery of Wannier-Mott excitons: E.F. Gross, 1956

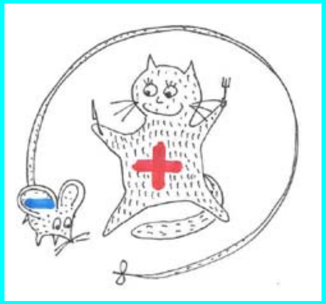


$$E_n = 2{,}17244 - \frac{0{,}0972}{n_0^2}; \quad n_0 = 2,3,\ldots.$$

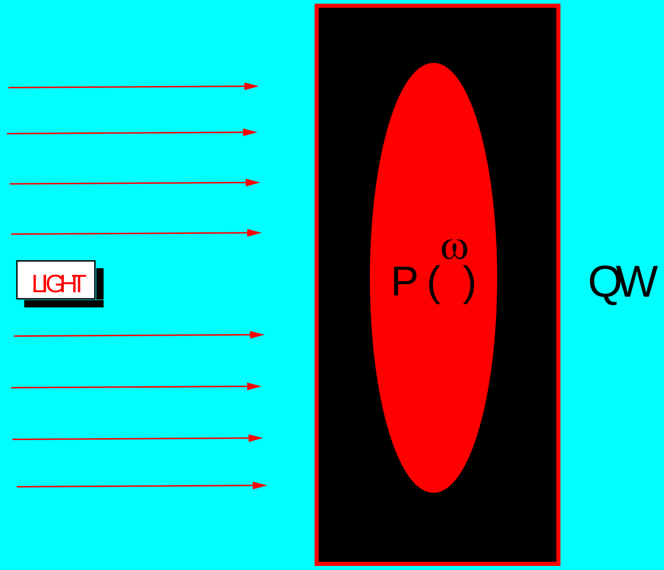
Exciton-polaritons: mixed exciton-photon states

Virtual chain of emission-absorption acts





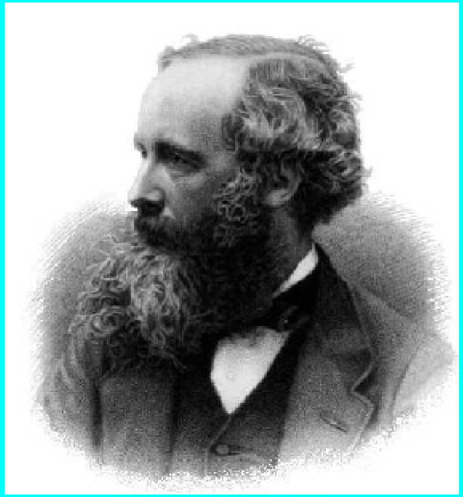
(quantum optics)



(semi-classical approach)

Time- and space-dependent dielectric polarization

LIGHT-MATTER COUPLING IN SOLIDS



Maxwell equations

$$\nabla \cdot \mathbf{D} = \frac{\rho}{\varepsilon_0}\,, \qquad \nabla \cdot \mathbf{B} = 0\,,$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{B}\,, \qquad \nabla \times \mathbf{B} = \frac{1}{\varepsilon_0 c^2}\partial_t \mathbf{J} + \frac{1}{c^2}\partial_t \mathbf{D}\,,$$

Electric displacement field $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \boldsymbol{\varepsilon}\mathbf{E}$

and Lorentz oscillator model

$$m_0 \ddot{x} + m_0 2\gamma \dot{x} + m_0 \omega_0^2 x = -eE(t)$$

damping

potential

driving force

Solution: $x(t \to \infty) = \mathcal{A}\cos(\omega t - \phi)$

$$\mathcal{A}(\omega) = \frac{-eE_0}{m_0}\frac{1}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\gamma\omega)^2}} \qquad \phi(\omega) = \arctan\left(\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)$$

Hopfield, J. J. (1958). Theory of the contribution of excitons to the complex dielectric constant of crystals, *Phys. Rev.* **112**: 1555.

Hopfield equations

$$\nabla \cdot \mathbf{D} = \cancel{\frac{\rho}{\varepsilon_0}},$$

$$\nabla \cdot \mathbf{B} = 0\,,$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\partial_t \mathbf{B}\,,$$

$$\nabla \times \mathbf{B} = \cancel{\frac{1}{\varepsilon_0 c^2}\partial_t \mathbf{J}} + \frac{1}{c^2}\partial_t \mathbf{D}$$

Maxwell equations

Displacement field

$$\mathbf{D} = \varepsilon_{\mathrm{B}} \mathbf{E} + \mathbf{P} = \boldsymbol{\varepsilon} \mathbf{E}$$

$$\frac{\varepsilon_{\mathrm{B}}}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) + \nabla \times \nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}(\mathbf{r},t)$$

1st Hopfield equation

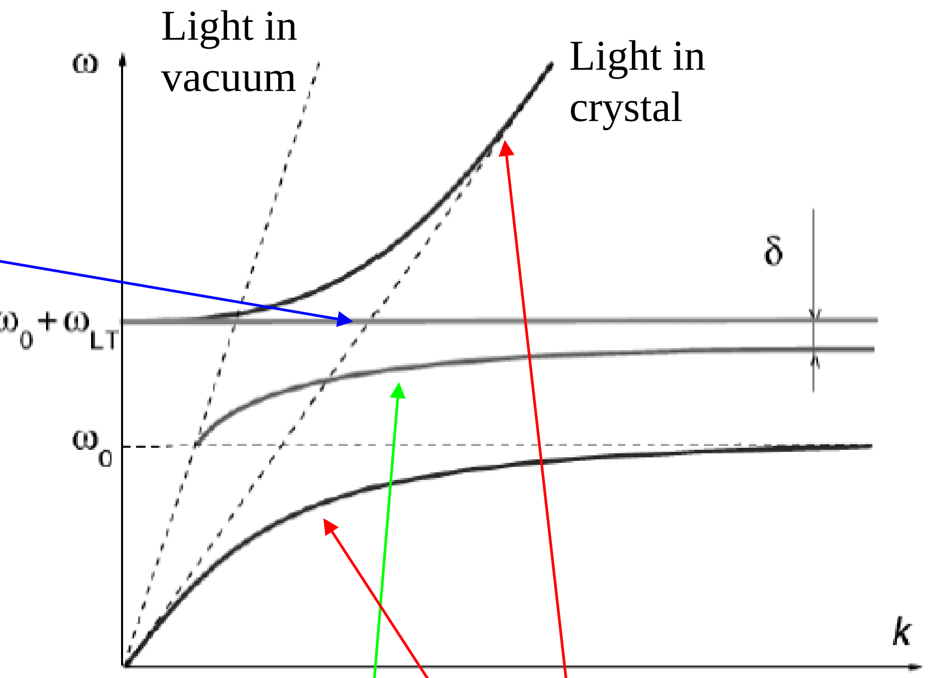
Lorentz oscillator

$$m_0\ddot{x} + m_0 2\gamma\dot{x} + m_0\omega_0^2 x = -eE(t)$$

$$\left[\frac{\partial^2}{\partial t^2} + 2\gamma\frac{\partial}{\partial t} + \omega_0^2 - \frac{\hbar\omega_0}{M_x}\nabla^2\right]\mathbf{P}(\mathbf{r},t) = \varepsilon_{\mathrm{B}}\omega_{\mathrm{p}}^2\mathbf{E}(\mathbf{r},t)$$

2nd Hopfield equation

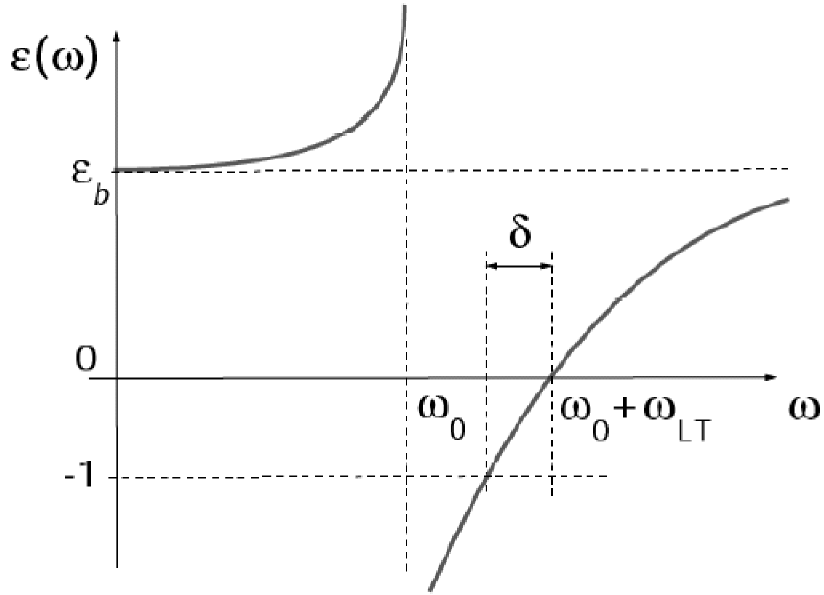
Dispersion of bulk exciton-polaritons in the limit $M_x \to \infty \quad \gamma \to 0$



Longitudinal polariton

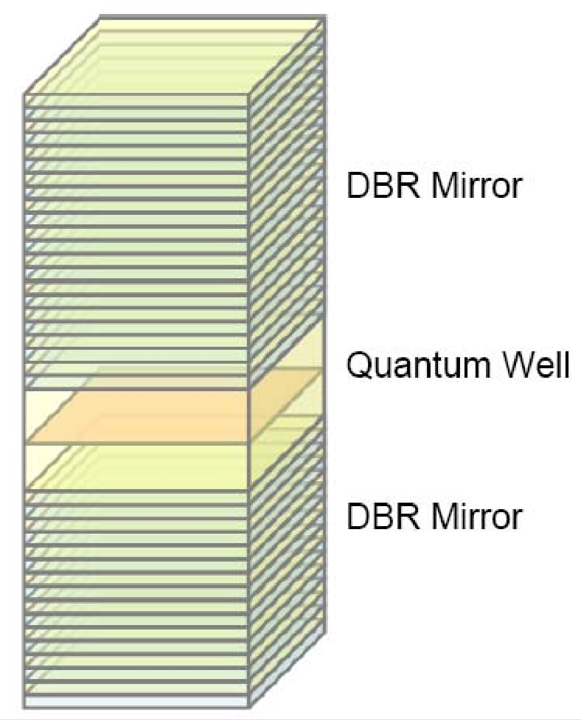
Transverse polaritons

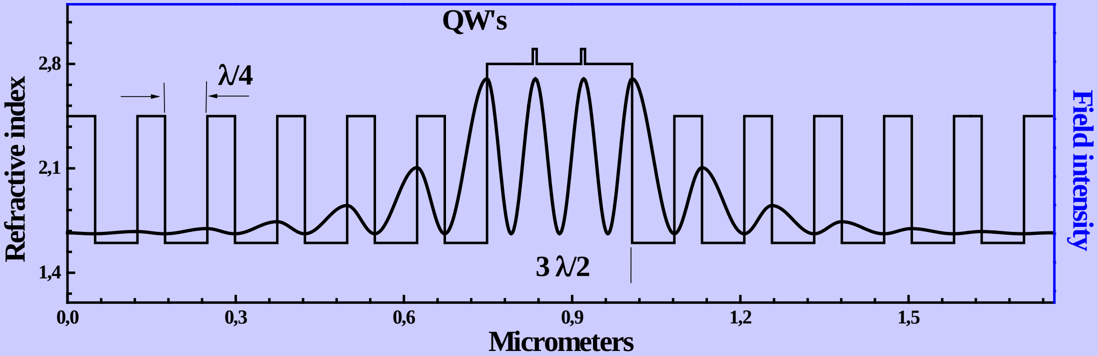
Surface polariton

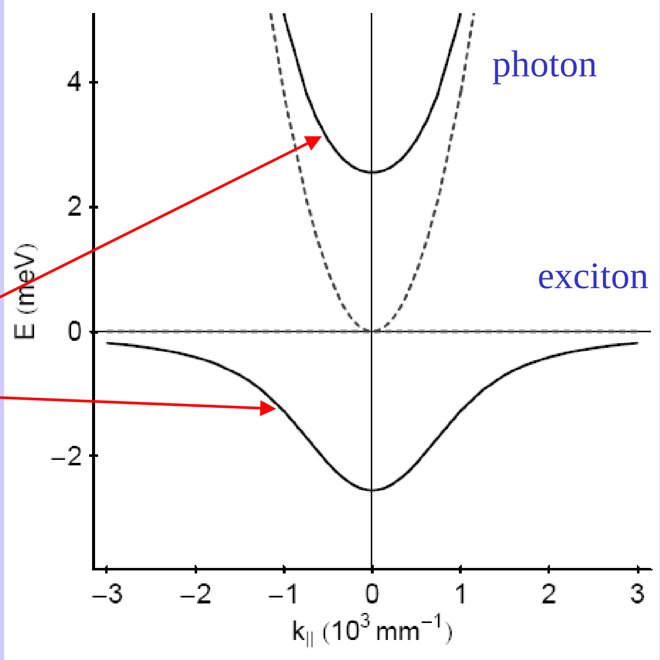


Part 4 Polaritonics

Polaritons in microcavities







Exciton polaritons

Concept of polariton lasing:

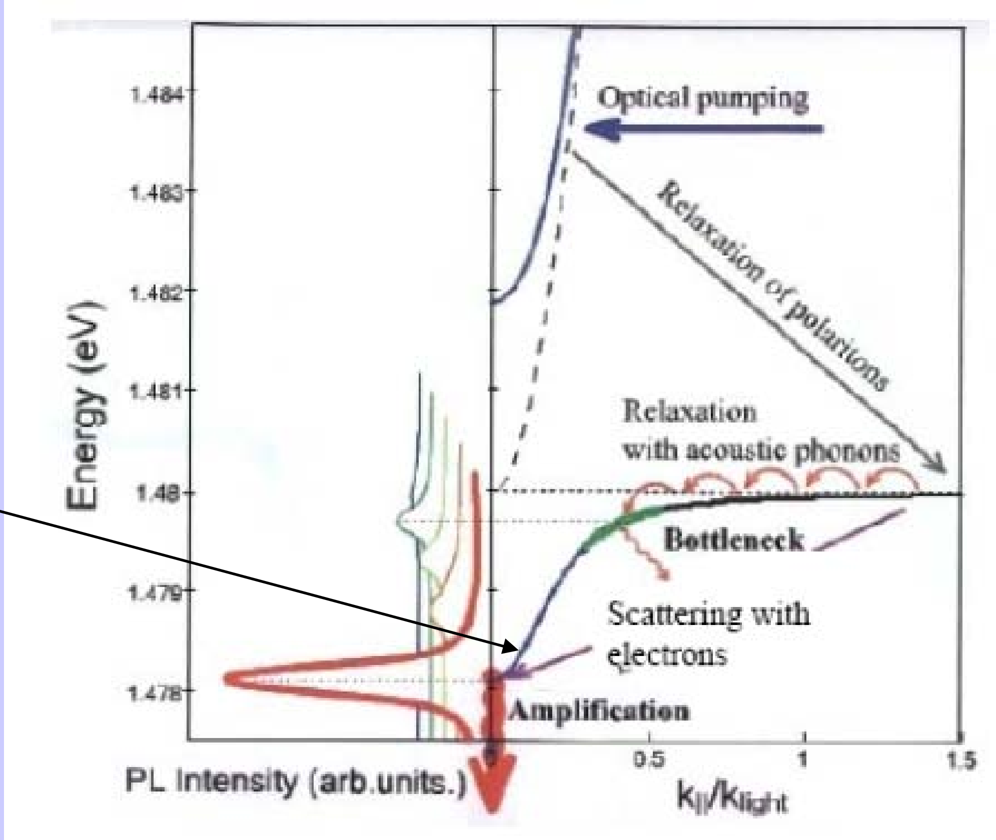
A. Imamoglu, et al, Phys. Lett. A 214, 193 (1996).

Photon mode dispersion

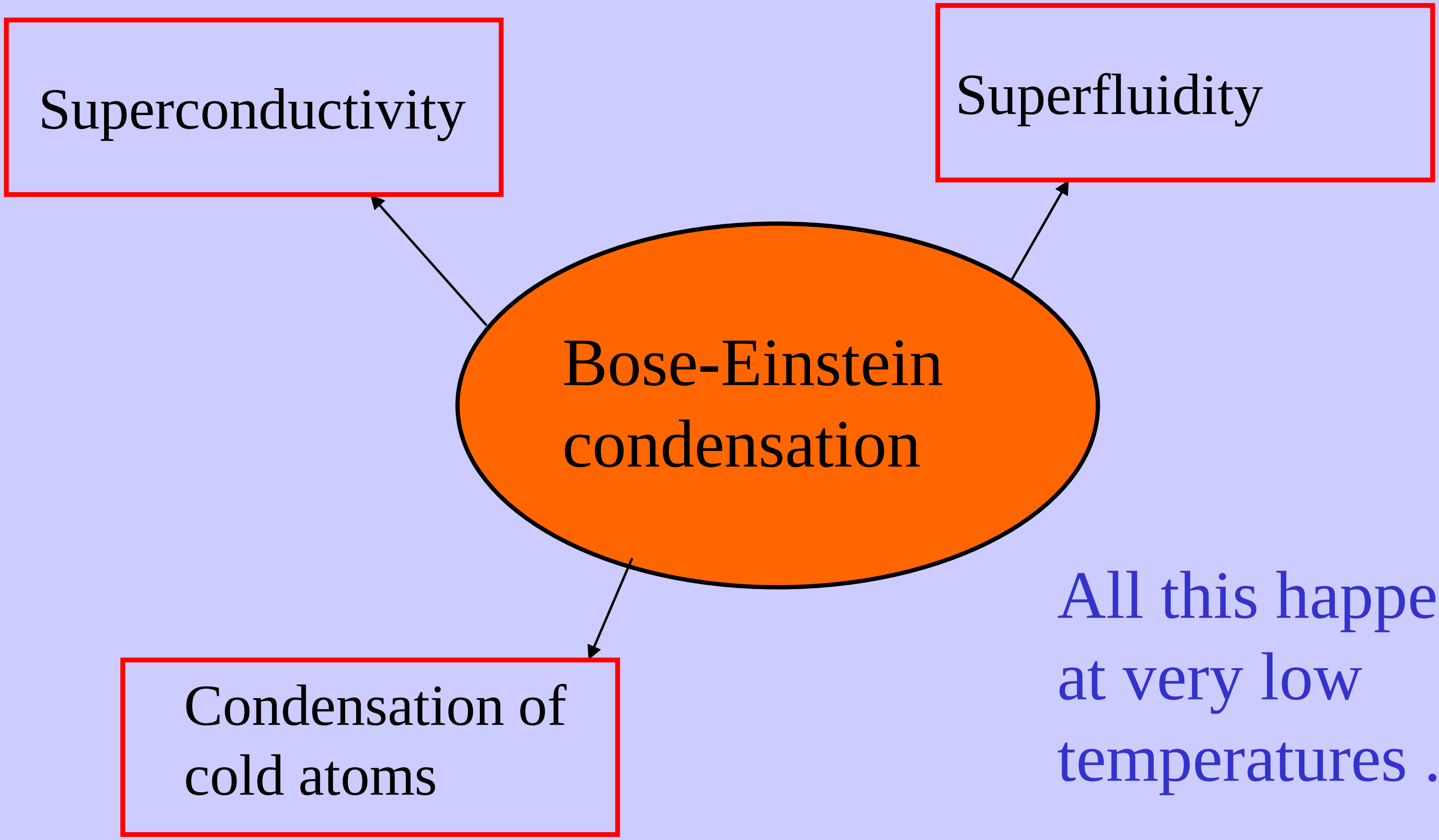
$$\frac{\omega}{c} n = \sqrt{\left(\frac{2\pi}{L}\right)^2 + k_\parallel^2}$$

Extremely light effective mass

$$\propto \left(10^{-5} - 10^{-4}\right) m_0$$



Optically or electronically excited exciton-polaritons relax towards the ground state and Bose-condense there. Their relaxation is stimulated by final state population. The condensate emits spontaneously a coherent light



Superconductivity

Superfluidity

Bose-Einstein condensation

Condensation of cold atoms

All this happens at very low temperatures ...

Exciton-polaritons: very light effective mass $\Rightarrow$ very high crytical temperature for BEC!

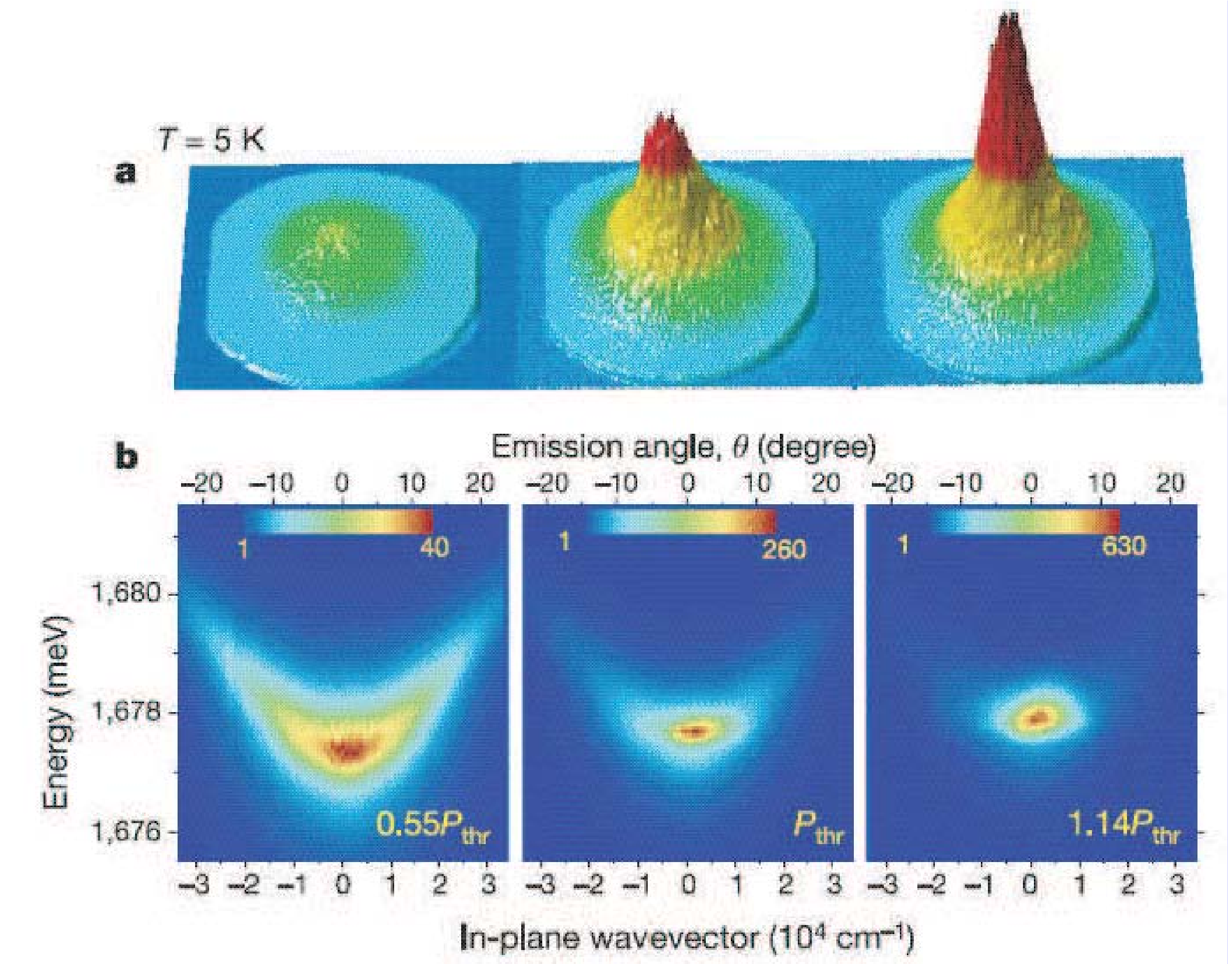
Vol 443|28 September 2006|doi:10.1038/nature05131

nature

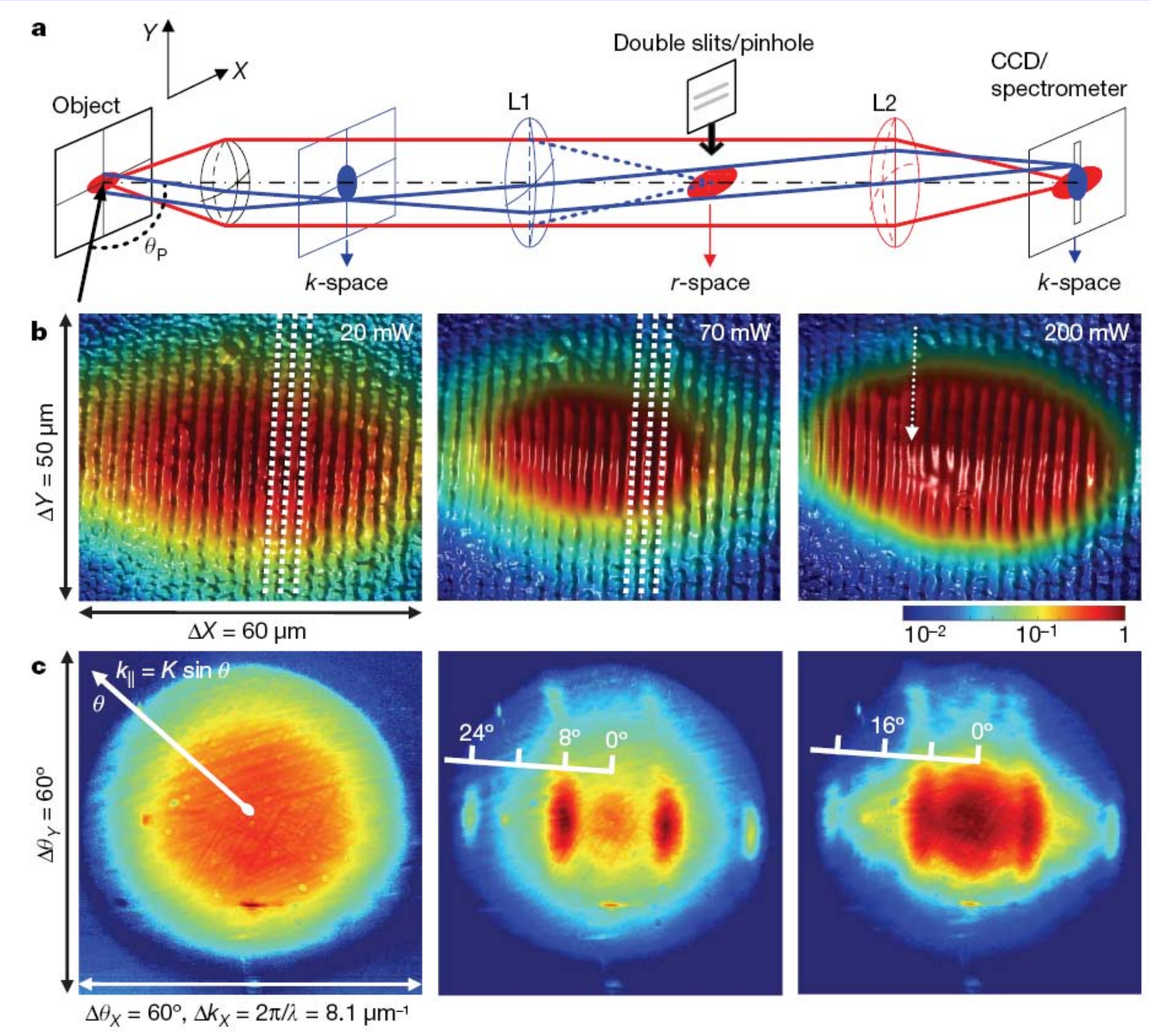
ARTICLES

Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]



Images of polariton condensates in real space



$T<T_c$

$T>T_c$

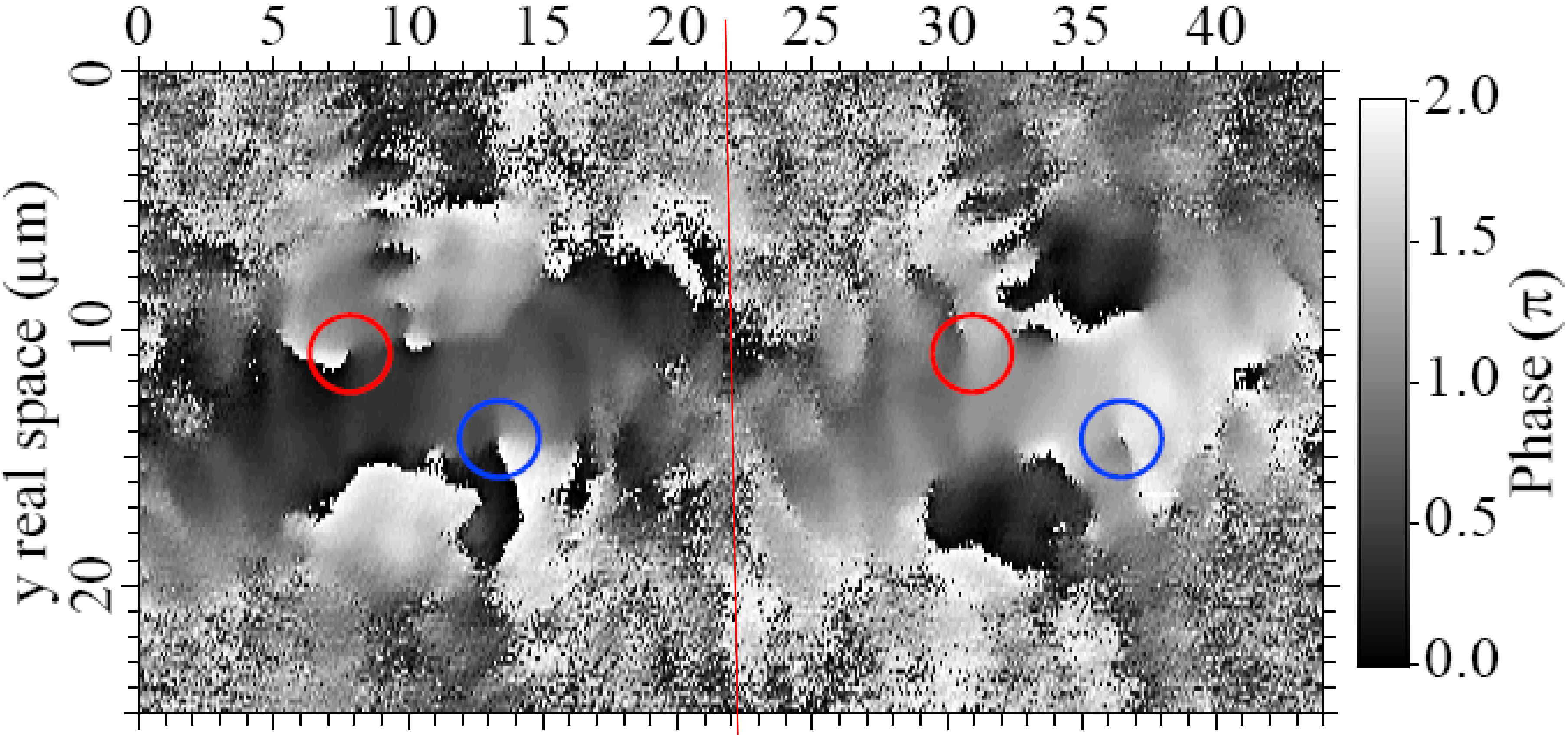
Metallic stripes on the top of the cavity create a superlattice potential

C.W. Lai *et al.*, *Nature* **450**, 529 (2007)

Science **326**, 974 (2009);
DOI: 10.1126/science.1177980

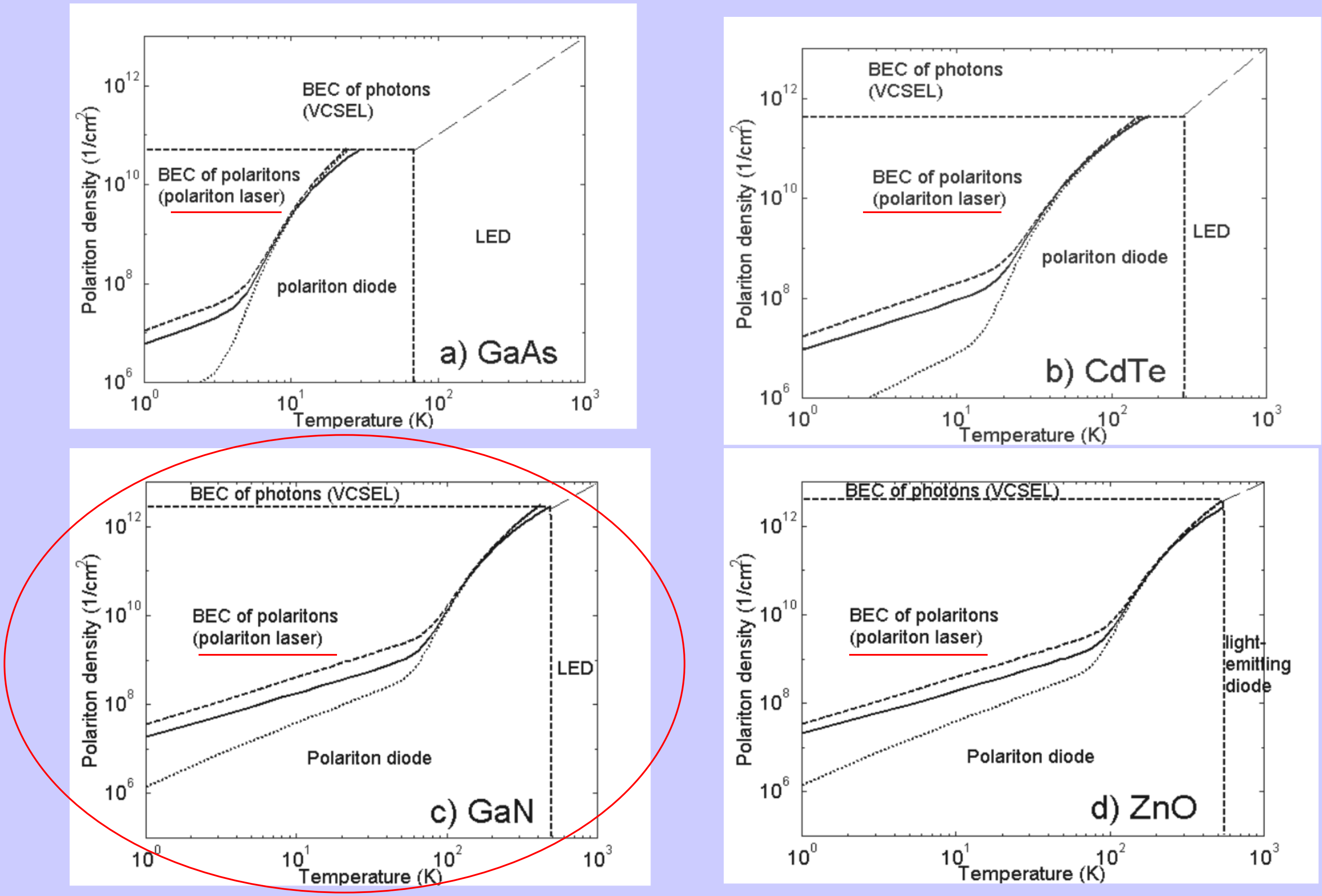
Observation of Half-Quantum Vortices in an Exciton-Polariton Condensate

K. G. Lagoudakis,[1*] T. Ostatnický,[2] A. V. Kavokin,[2,3] Y. G. Rubo,[4] R. André,[5] B. Deveaud-Plédran[1]



Phase map of a microcavity sample

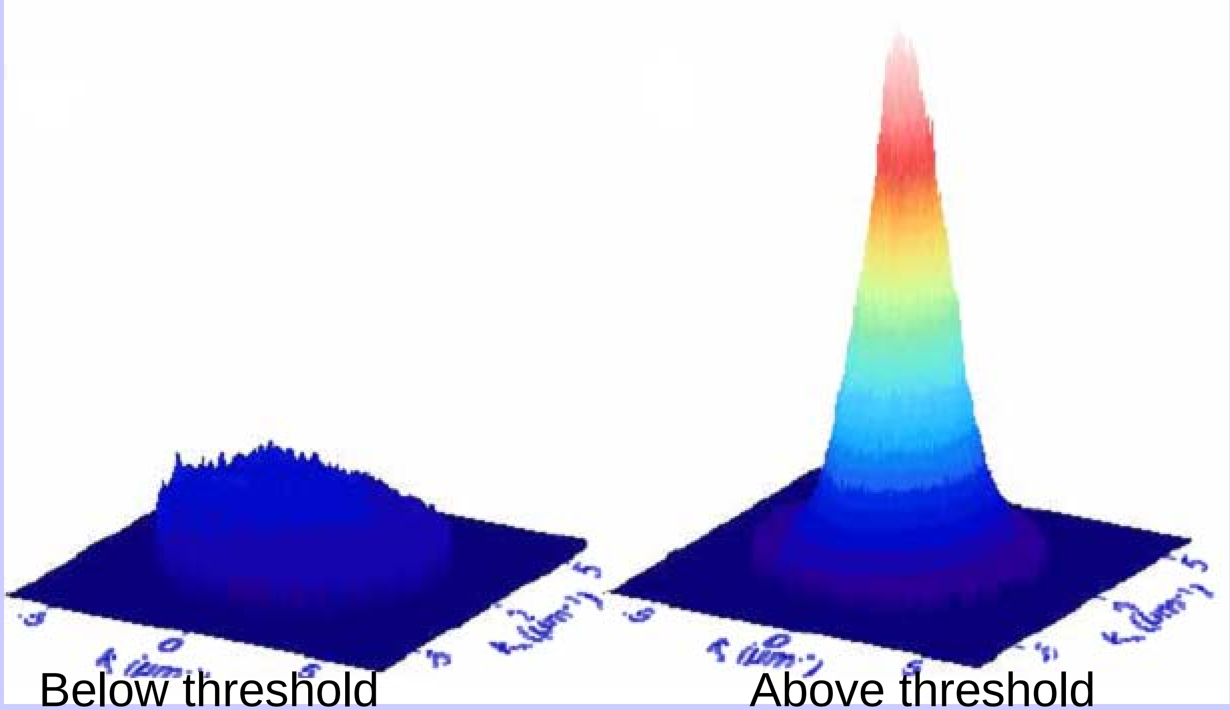
Phase diagrams for BEC of exciton-polaritons in different model cavities



Solid lines show the critical concentration N_c versus temperature of the polariton KT phase transition. Dotted and dashed lines show the critical concentration N_c for quasi condensation in 100 μm and 1 meter lateral size systems, respectively.

Kavokin A, Malpuech G, Gil B, Semiconductor microcavities: towards polariton lasers, MRS Internet Journal of Nitride Semiconductor Research ***8*** *(3): 3 (2003)*

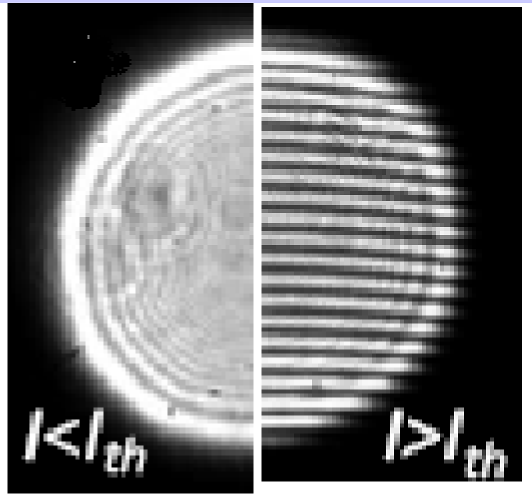
Build-up of the condensate in a GaN microcavity at 300 K



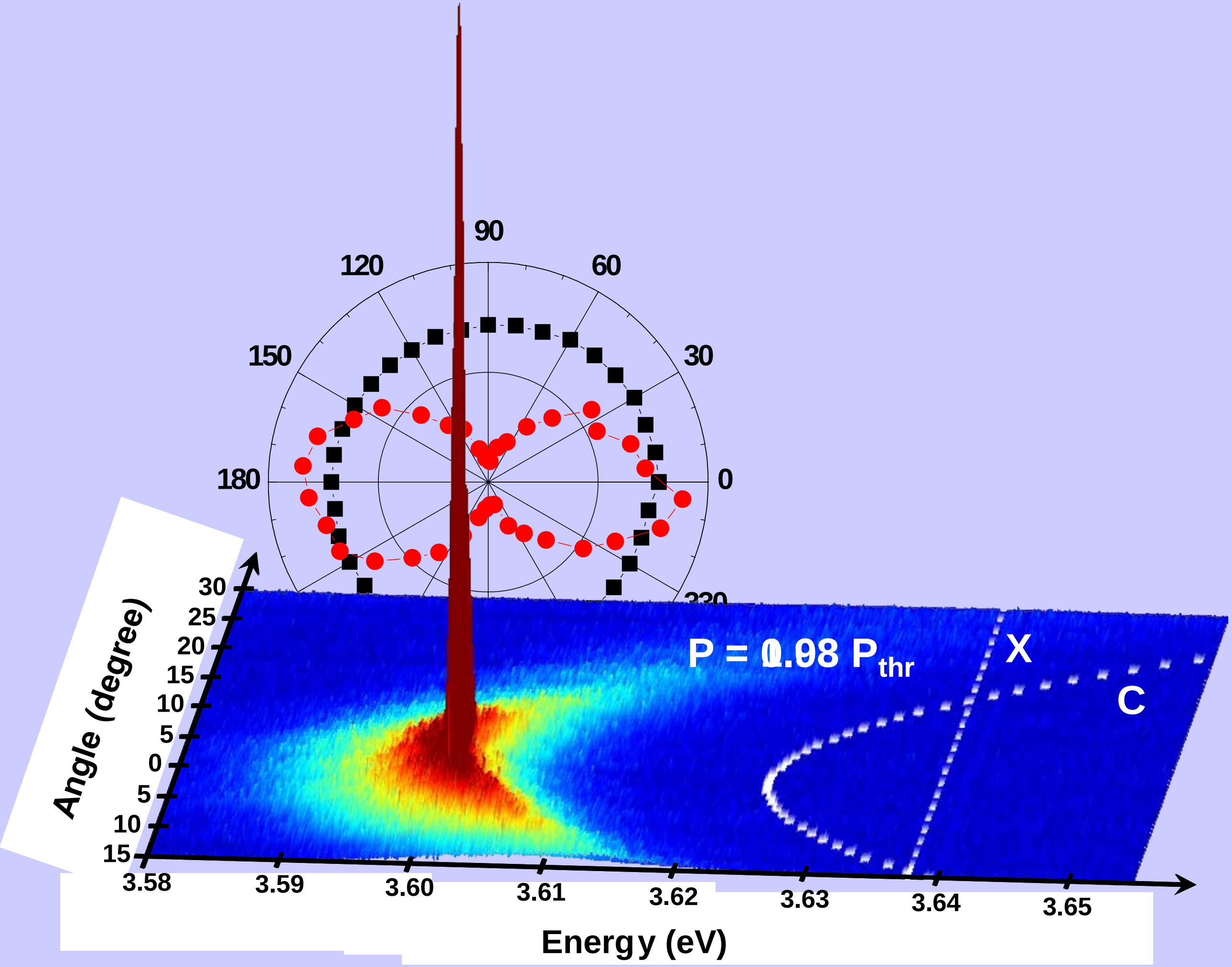
Below threshold

Above threshold

J. J. Baumberg, A. V. Kavokin, S. Christopoulos, A. J. Grundy, R. Butté, G. Christmann, D. D. Solnyshkov, G. Malpuech, G. Baldassarri Höger von Högersthal, E. Feltin, J.-F. Carlin, and N. Grandjean, Spontaneous Polarization Buildup in a Room-Temperature Polariton Laser, **Phys. Rev. Lett. 101**, 136409 (2008).



Room temperature polariton lasing



Nonlinear emission peaked at $k_{//} = 0$

Linearly polarized emission above threshold (polarization degree: 80%)

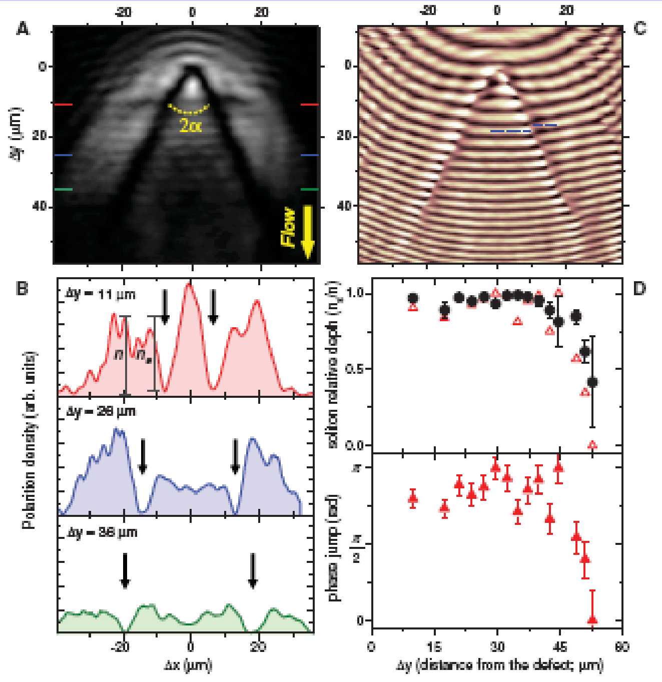
Emission still in the strong coupling regime

Polarization splitting: ~150 μeV

G. Christmann *et al.*, Appl. Phys. Lett. **93**, 051102 (2008)

POLARITONICS in 2011 (recent achievements)

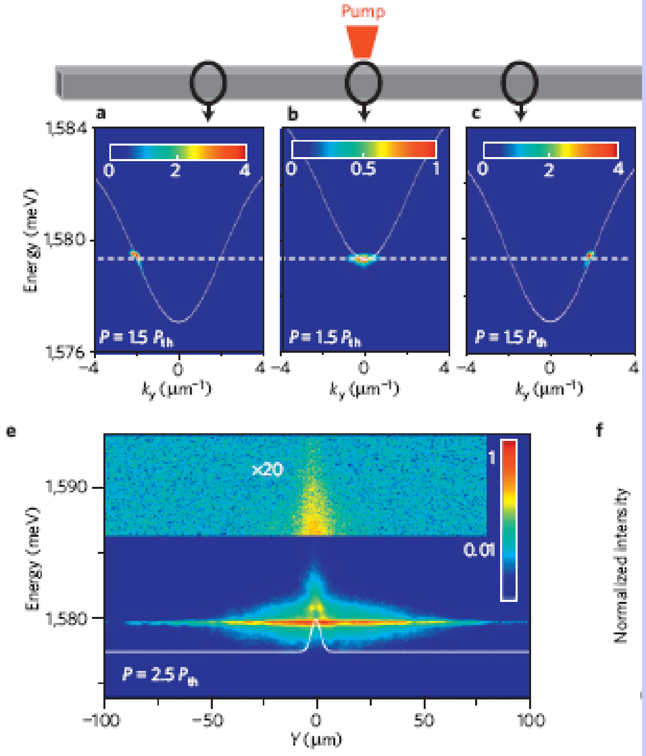
Superfluidity, vortices, solitons...



Involved: LKB, LPN, Madrid, Sheffield, Cambridge

Publications in Science, Nature, Nature Physics, Nature Photonics 2009-2011

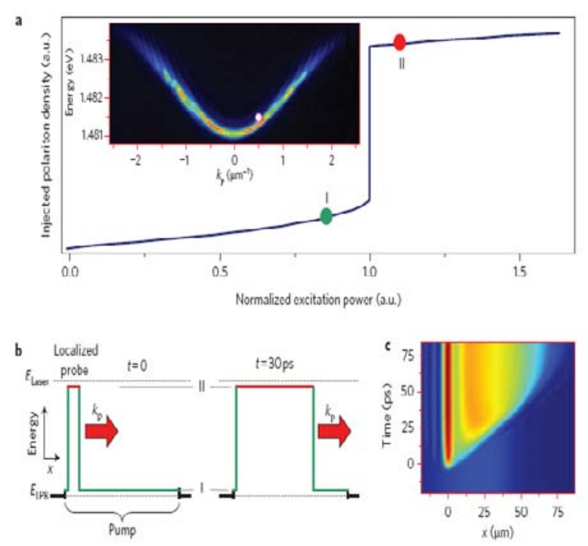
Polariton condensation in stripes



LPN, Southamoton, Clermont-Fd

Nature Physics 2010

Polariton spin switches



LKB, Southampton, EPFL

Nature Photonics 2010

PHYSICAL REVIEW B VOLUME 7, NUMBER 3 1 FEBRUARY 1973

Model for an Exciton Mechanism of Superconductivity*

David Allender,† James Bray, and John Bardeen
epartment of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801

(Received 7 August 1972)

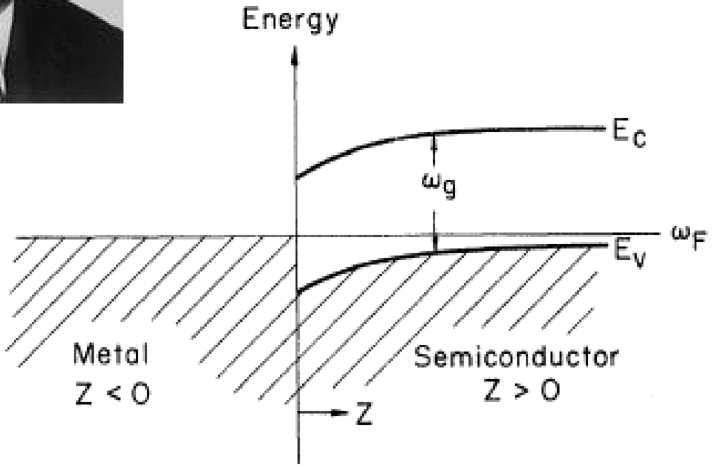


FIG. 1. Metal-semiconductor interface. E_c and E_v are the bottom of the conduction band and top of the valence band, respectively.

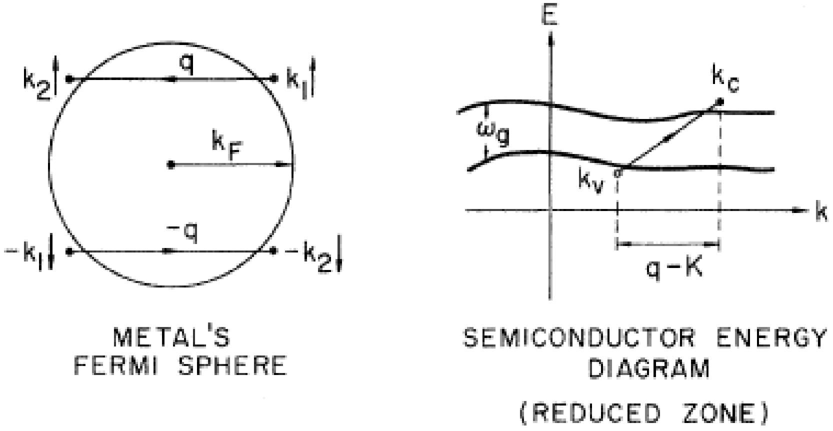
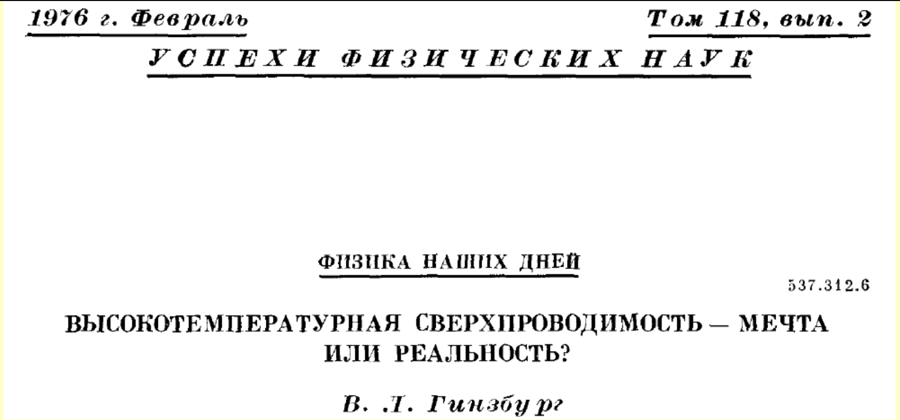


FIG. 4. Illustration of the exciton scattering process.

illustrated in Fig. 4. A metal electron $\vec{k}_1\uparrow$ is shown to scatter to $\vec{k}_2\uparrow$ by exciting a semiconductor valence-band electron k_v into a state above the gap $\vec{k}_c$ and creating a virtual exciton. The paired electron $-\vec{k}_1\downarrow$ then absorbs the exciton and scatters into the state $-\vec{k}_2\downarrow$. Conservation of wave vector requires that $\vec{q}=\vec{k}_2-\vec{k}_1=\vec{k}_v-\vec{k}_c+\vec{K}$, where $\vec{K}$ is a reciprocal lattice vector. In general, $\vec{q}$ may lie outside of the first Brillouin zone, and thus there are several values of $\vec{K}$ and of $\vec{k}_v-\vec{k}_c$ that satisfy this condition. We stress that the exciton scattering is conceptually identical to the scattering of electrons though the exchange of phonons.

Part 5 Excitons and Superconductivity



1976 г. Февраль Том 118, вып. 2

УСПЕХИ ФИЗИЧЕСКИХ НАУК

ФИЗИКА НАШИХ ДНЕЙ

537.312.6

ВЫСОКОТЕМПЕРАТУРНАЯ СВЕРХПРОВОДИМОСТЬ — МЕЧТА ИЛИ РЕАЛЬНОСТЬ?

В. Л. Гинзбург

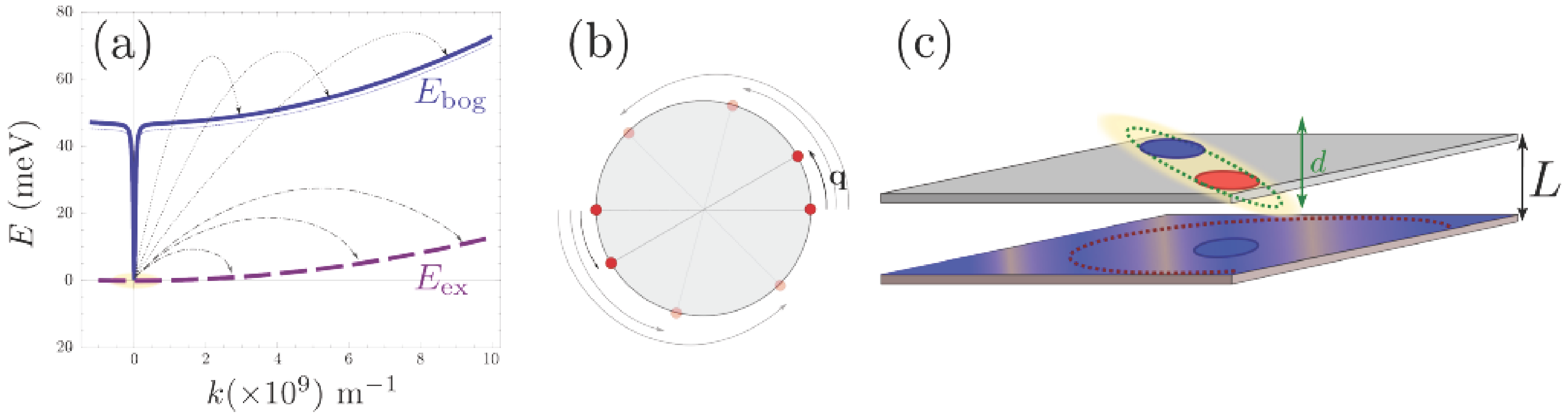
- An exciton mechanism may be realised in 2D metal-dielectric sandwiches (higher λ).
- Non-equilibrium superconductivity has a great future

BUT IT NEVER WORKED ! WHY ?

1) Exciton-electron interaction still weak;
2) Excitons are too fast (reduced retardation effect), consequently:
3) Coulomb repulsion becomes important.

We consider the following model structures:

a heavily n-doped layer embedded between two neutral QWs in a microcavity



or a thin layer of metal on the top of biased coupled quantum wells

F.P. Laussy, A.V. Kavokin and I.A. Shelykh, Exciton polariton mediated superconductivity, Physical Review Letters, **104**, 106402 (2010).

F.P. Laussy, T. Taylor, I.A. Shelykh, A.V. Kavokin, Superconductivity with excitons and polaritons, unpublished

Electrons + exciton-polariton BEC: interaction Hamiltonian

Coulomb repulsion

$$H = \sum_{\mathbf{k}} \left[E_{\mathrm{el}}(\mathbf{k}) \sigma^{\dagger}_{\mathbf{k}} \sigma_{\mathbf{k}} + E_{\mathrm{pol}}(\mathbf{k}) a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} \right] +$$

$$+ \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} \left[V_{\mathrm{C}}(\mathbf{q}) \sigma^{\dagger}_{\mathbf{k}_1+\mathbf{q}} \sigma^{\dagger}_{\mathbf{k}_2-\mathbf{q}} \sigma_{\mathbf{k}_1} \sigma_{\mathbf{k}_2}, \right.$$

$$\left. + X V_{\mathrm{X}}(\mathbf{q}) \sigma^{\dagger}_{\mathbf{k}_1} \sigma_{\mathbf{k}_1+\mathbf{q}} a^{\dagger}_{\mathbf{k}_2+\mathbf{q}} a_{\mathbf{k}_2} + U a^{\dagger}_{\mathbf{k}_1} a^{\dagger}_{\mathbf{k}_2+\mathbf{q}} a_{\mathbf{k}_1+\mathbf{q}} a_{\mathbf{k}_2} \right]$$

Electron-polariton interactions

Polariton-polariton interactions

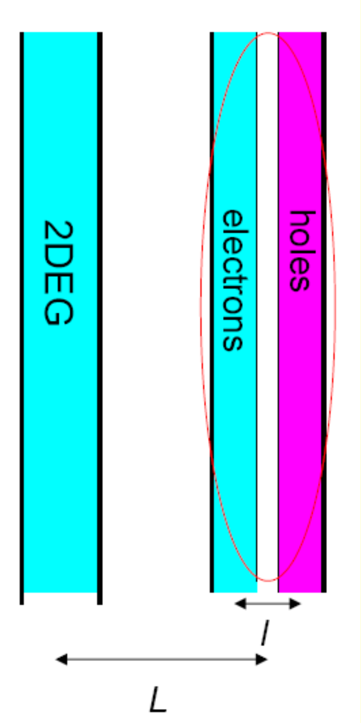
Interactions:

Electron-exciton interaction:

$$V_{dir}(q) = \frac{e^2}{2\epsilon_0\epsilon A}\frac{e^{-qL}}{q}\left\{\frac{e^{\beta_e ql}}{[1+(\beta_e q a_B/2)^2]^{3/2}} - \frac{e^{-\beta_h ql}}{[1+(\beta_h q a_B/2)^2]^{3/2}}\right\}$$

$$\beta_e = \frac{m_e}{m_e + m_h}$$

$$\beta_h = \frac{m_h}{m_e + m_h}$$



Electron-electron interaction:

$$V_C(\mathbf{q}) = e^2/[2\epsilon A(|\mathbf{q}| + \kappa)]$$

Boglyubov transformation:

$$a_q = \frac{1}{\sqrt{1-A_q^2}}\left(b_q + A_q b^+_{-q}\right)$$

$$a^+_{-q} = \frac{1}{\sqrt{1-A_q^2}}\left(b^+_{-q} + A_q b_q\right)$$

$$a^+_{k_1+q} a_{k_1} \approx \langle a^+_{k_1+q}\rangle a_{k_1} + a^+_{k_1+q}\langle a_{k_1}\rangle = \sqrt{N_0}\left(\delta_{k_1+q} a_{-q} + \delta_{k_1} a^+_q\right)$$

$$\boxed{\hat{H} = \sum_k E_{el}(k) c^+_k c_k + \sum_k E_{bog}(k) b^+_k b_k + \sqrt{N_0}\sum_{k,q}\overline{M}(q) c^+_k c_{k+q}(b^+_{-q} + b_q)}$$

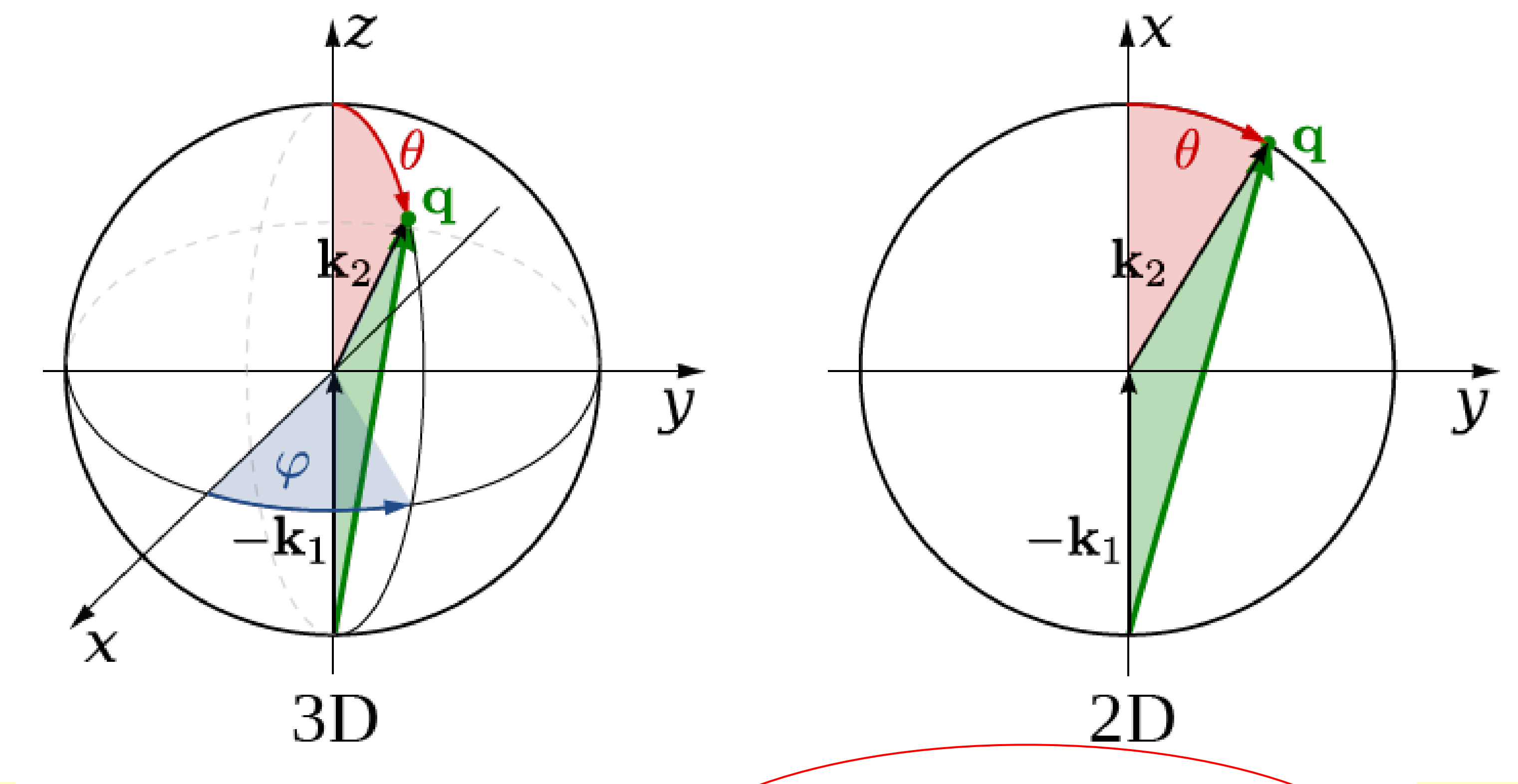
Concentration of exciton-polaritons

$$\overline{M}(q) = V(q)\sqrt{\frac{1+A_q}{1-A_q}}$$

$$A_q = U^{-1}\left[-E_{pol}(q) - UN_0 + \sqrt{(E_{pol}(q) + UN_0)^2 - (UN_0)^2}\right]$$

$$E_{\mathrm{bog}}(\mathbf{k}) = \sqrt{\tilde{E}_{\mathrm{pol}}(\mathbf{k})(\tilde{E}_{\mathrm{pol}}(\mathbf{k}) + 2UN_0A)}$$

Matrix element of electron-electron interaction



ed

$$M(\mathbf{q}) = \sqrt{N_0 A} X V_X(\mathbf{q}) \boxed{\sqrt{\frac{E_{\mathrm{bog}}(\mathbf{q}) - \tilde{E}_{\mathrm{pol}}(\mathbf{q})}{2UN_0A - E_{\mathrm{bog}}(\mathbf{q}) + \tilde{E}_{\mathrm{pol}}(\mathbf{q})}}} \xrightarrow{} 1.$$

$$q = \sqrt{2k_F^2(1 + \cos\theta)}$$

$$E_{\mathrm{bog}}(\mathbf{k}) = \sqrt{\tilde{E}_{\mathrm{pol}}(\mathbf{k})(\tilde{E}_{\mathrm{pol}}(\mathbf{k}) + 2UN_0A)}$$

Electron – electron interaction potential:

$$U_0(\omega) = \frac{A\mathcal{N}}{2\pi} \int_0^{2\pi} [V_A(q,\omega) + V_C(q)] d\theta,$$

$$V_A(\mathbf{q},\omega) = \frac{2M(\mathbf{q})^2 E_{\mathrm{bog}}(\mathbf{q})}{(\hbar\omega)^2 - E_{\mathrm{bog}}(\mathbf{q})^2}$$

← exciton mediated interaction

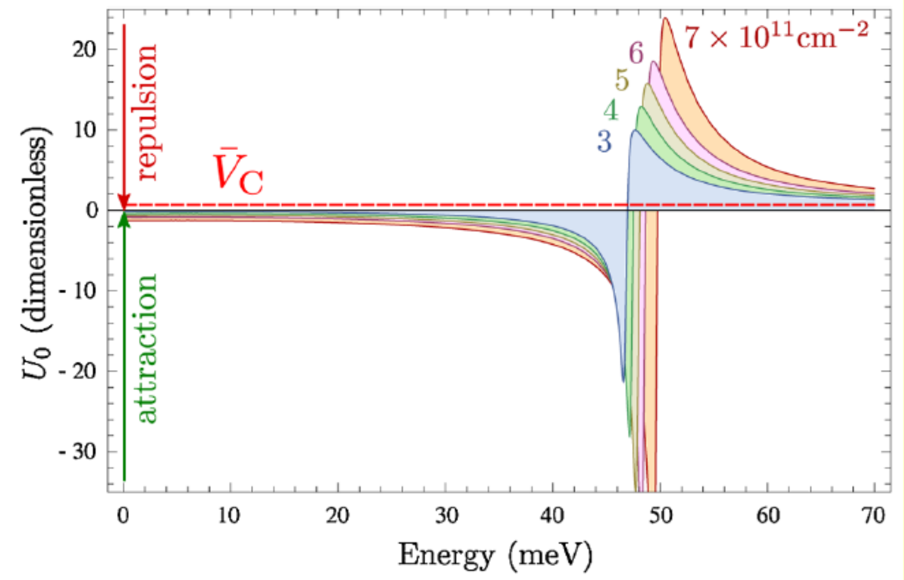
$$V_C(\mathbf{q}) = e^2/[2\epsilon A(|\mathbf{q}| + \kappa)]$$

← Coulomb repulsion

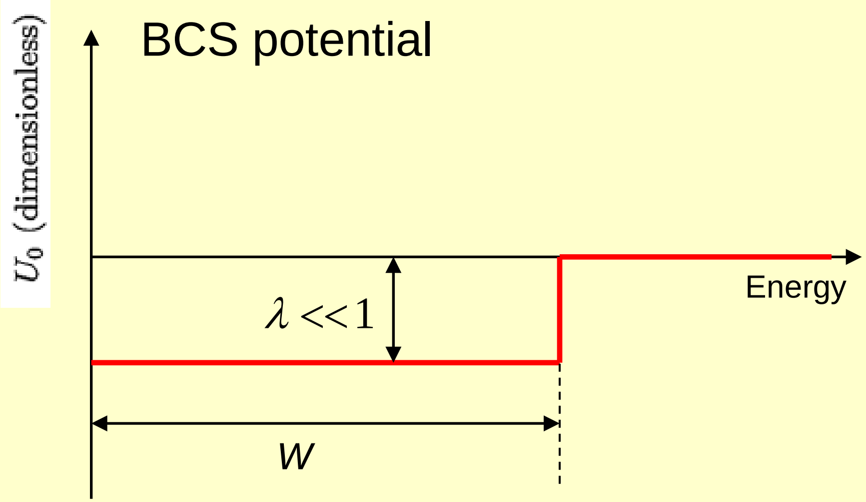
BCS neglected this term, but we cannot!

Our potential

$$U_0(\omega) = \frac{A\mathcal{N}}{2\pi}\int_0^{2\pi}[V_\mathrm{A}(q,\omega)+V_\mathrm{C}(q)]d\theta$$



BCS potential



Comparison with BCS

$$T_c = W\exp(-1/\lambda)$$

We have:

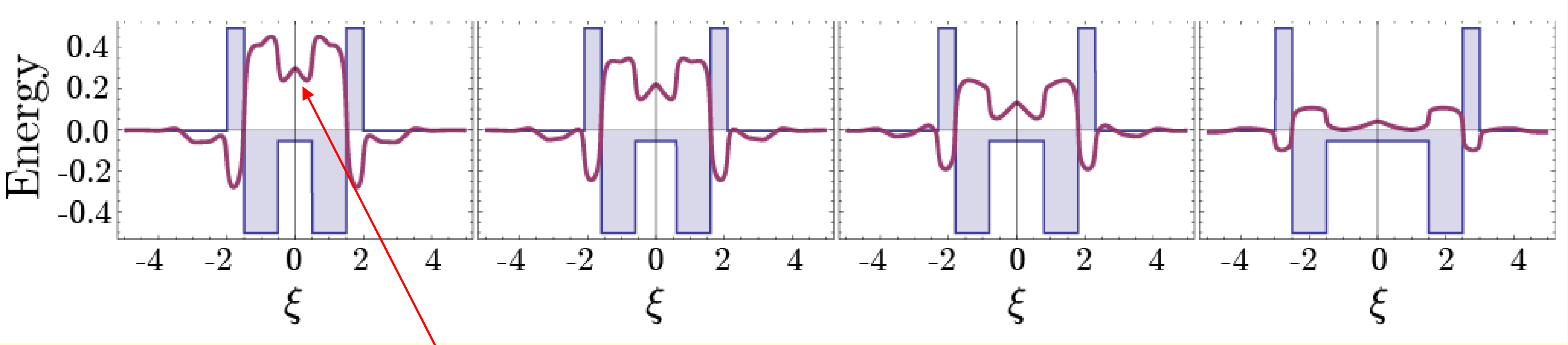
1) Much stronger attraction;
2) Similar Debye temperature
3) Peculiar shape of the potential

Solving the gap equation within the Bogolyubov approximation

$$\Delta(\xi,T) = -\int_{-\infty}^{+\infty} \frac{U_0(\xi-\xi')\Delta(\xi',T)\tanh(E/2k_{\mathrm{B}}T)}{2E}\,d\xi'$$

$$E = \sqrt{\Delta(\xi',T)^2 + \xi'^2}.$$

Model GaN cavity



we obtain the superconducting gap which vanishes at the critical temperature

Part 6 Realisation of exciton-mediated superconductivity

Indirect excitons or exciton-polaritons?

Huge dipole moment

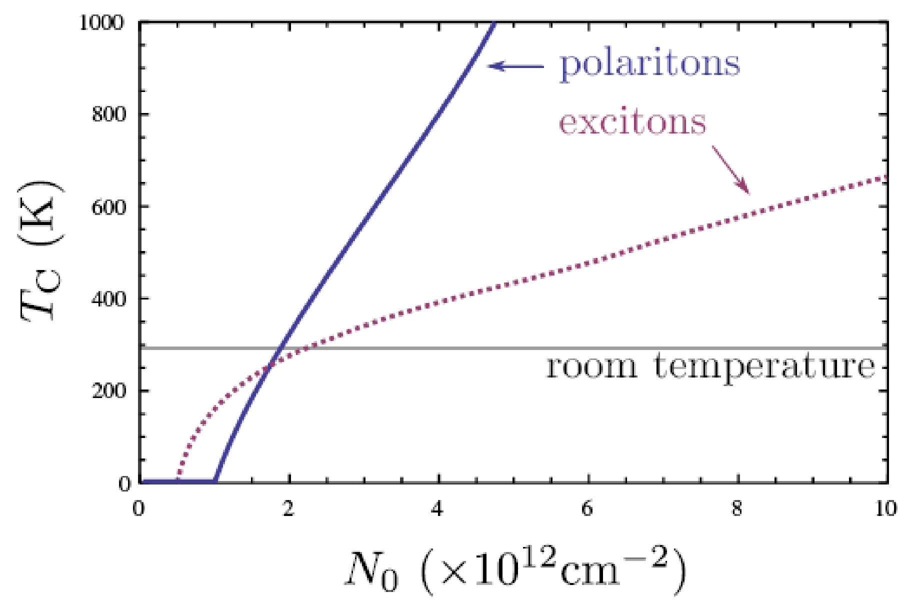
Long life time, low losses

Hard to have a condensate at high temperatures

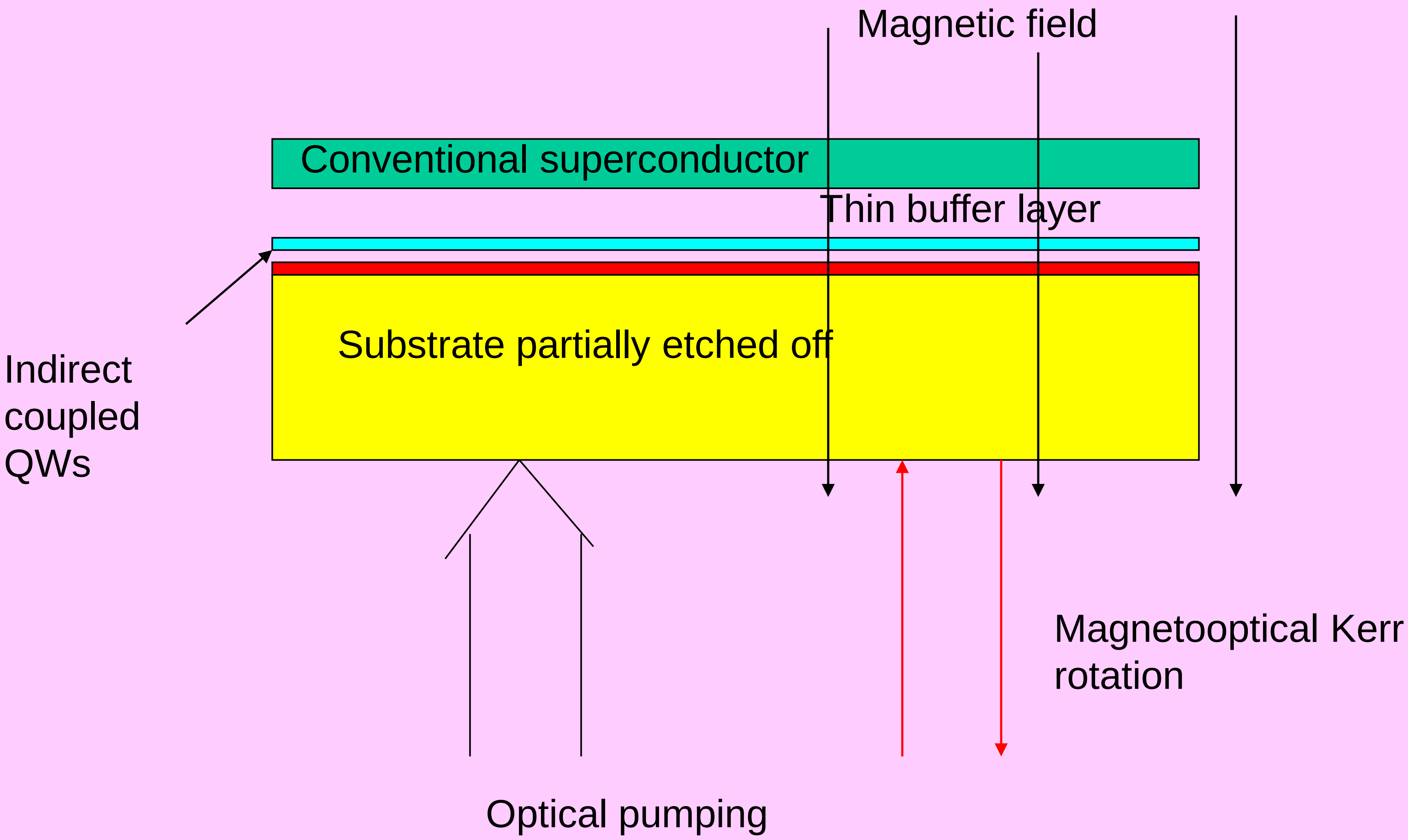
Difficult to measure conductivity in microcavities

Need of metal-semiconductor-dielectric structures

Direct optical control at high temperatures



How to detect the Optically induced superconductivity?



Magnetic field

Conventional superconductor

Thin buffer layer

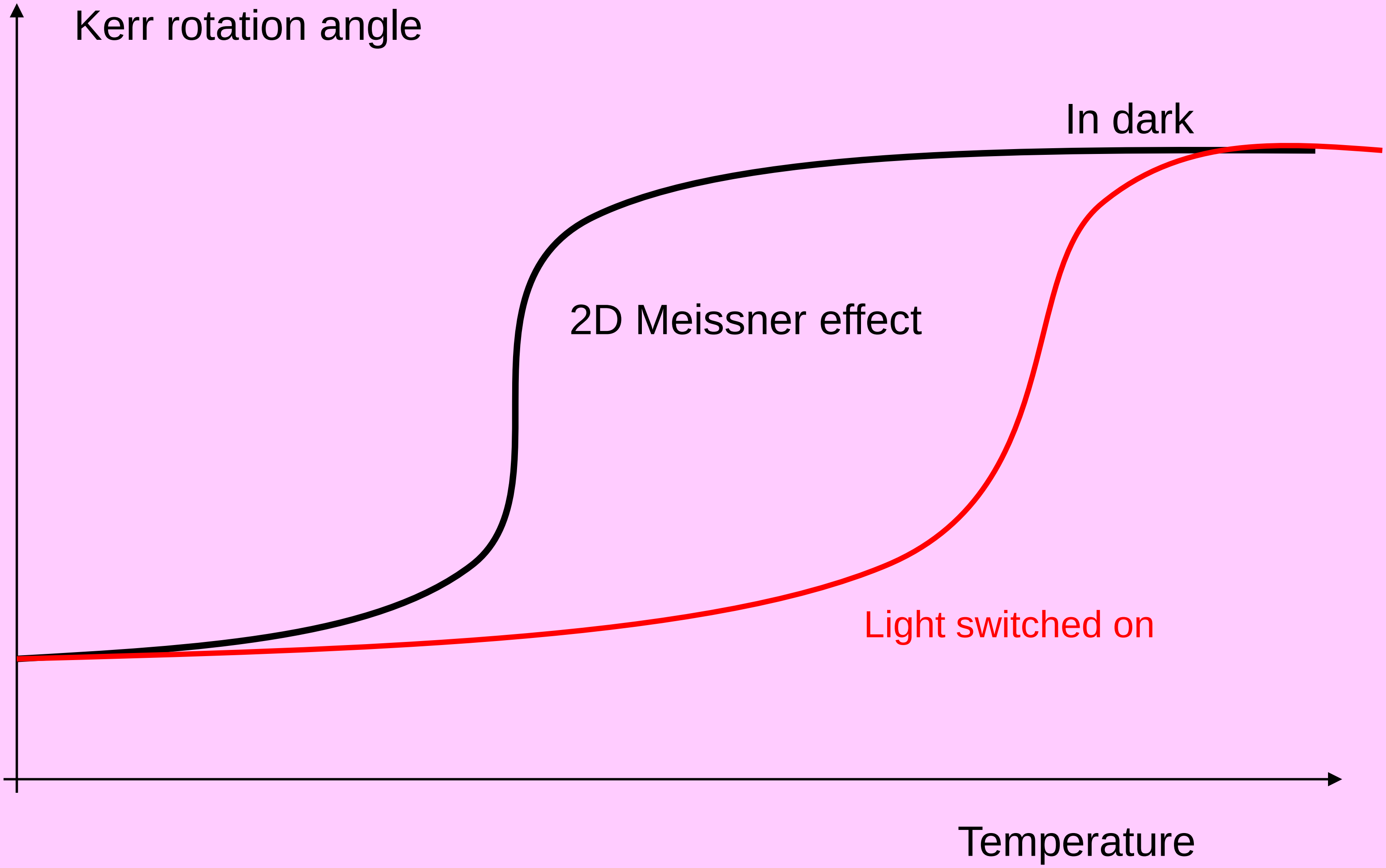
Indirect coupled QWs

Substrate partially etched off

Magnetooptical Kerr rotation

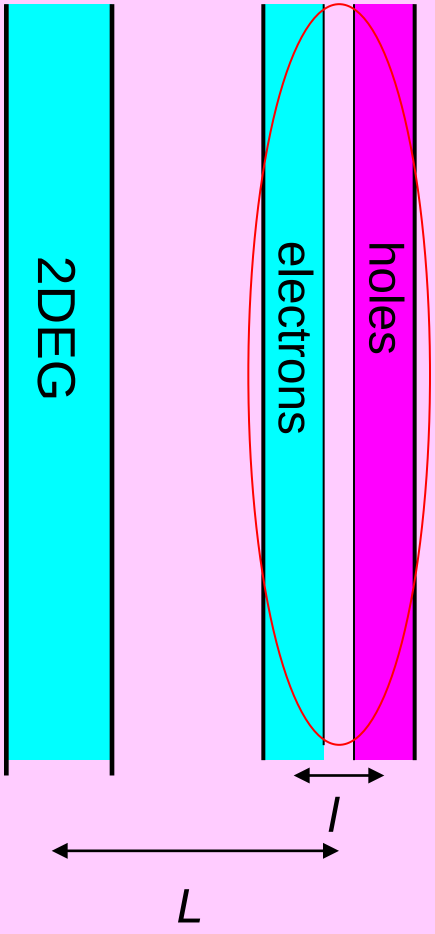
Optical pumping

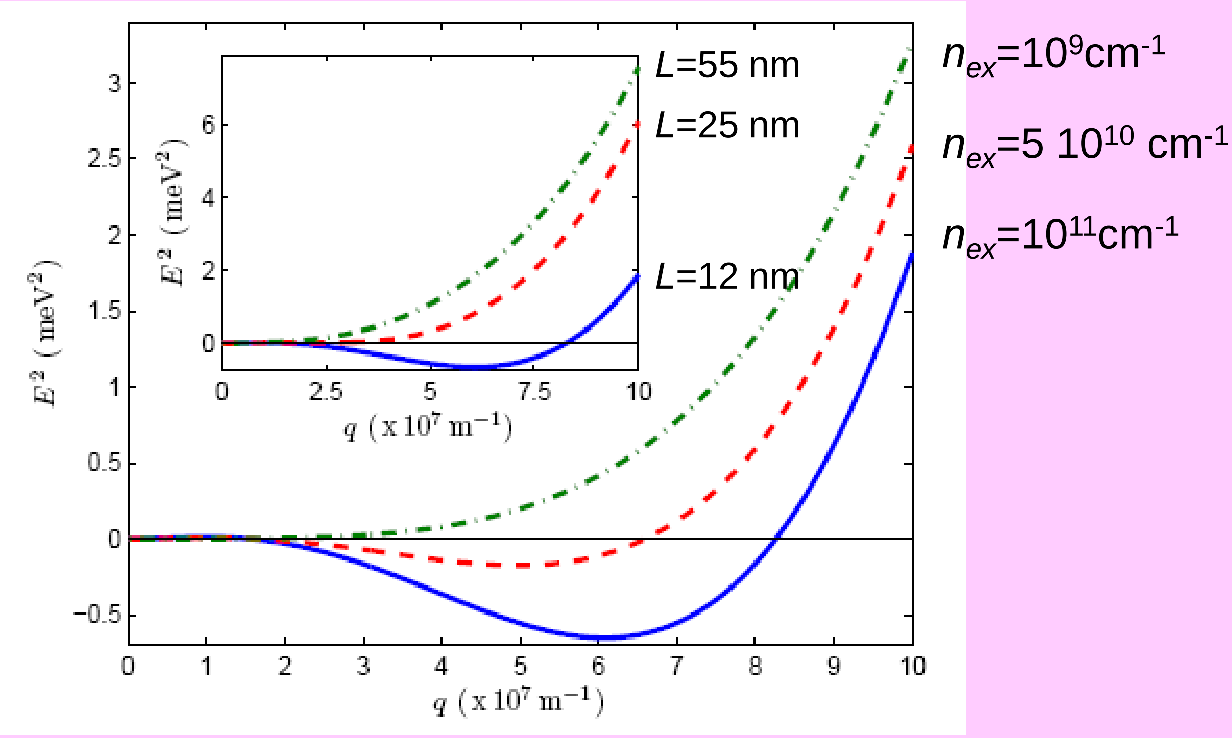
What should one try to observe:



Now we know what may happen to fermions,

But what will happen to bosons??





EXCITONIC SUPERSOLID !!

Rotons in a Hybrid Bose-Fermi System
Ivan A. Shelykh, Thomas Taylor, and Alexey V. Kavokin
Phys. Rev. Lett. **105**, 140402 (2010)

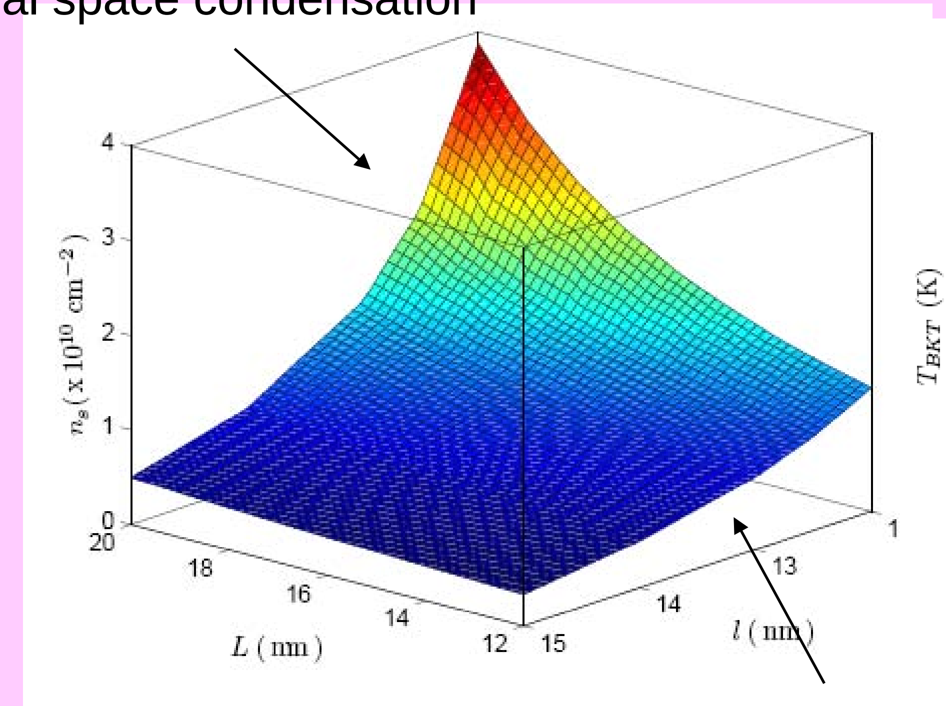
Suppression of the Bose-Einstein condensation and superfluidity

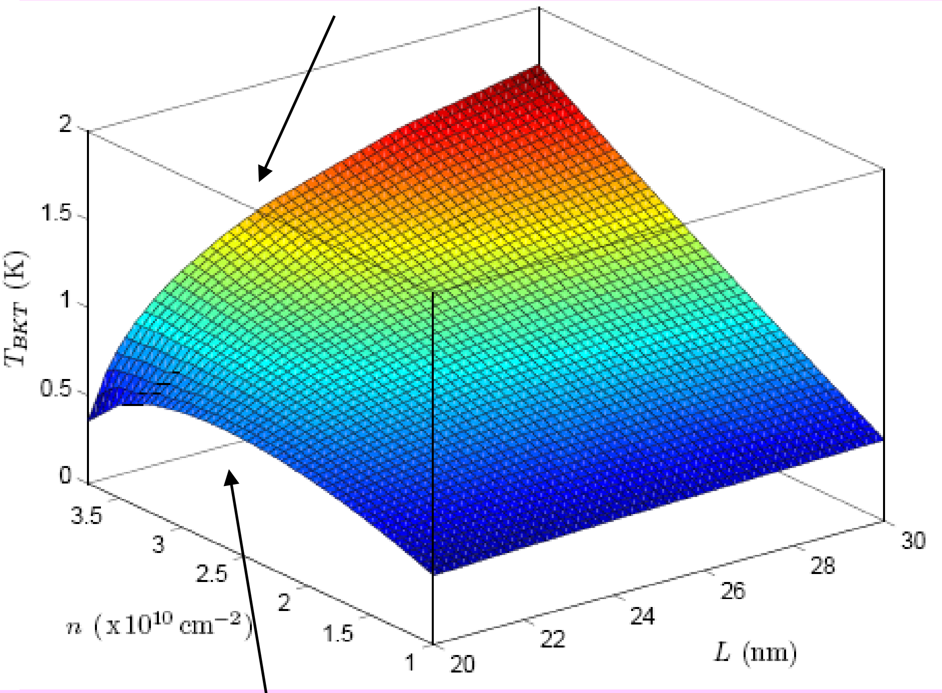
$$n_s = n - n_n(T_{BKT})$$

$$T_{BKT} = \frac{\pi\hbar^2 n_s(T_{BKT})}{2M}$$

real space condensation

classical fluid





BEC

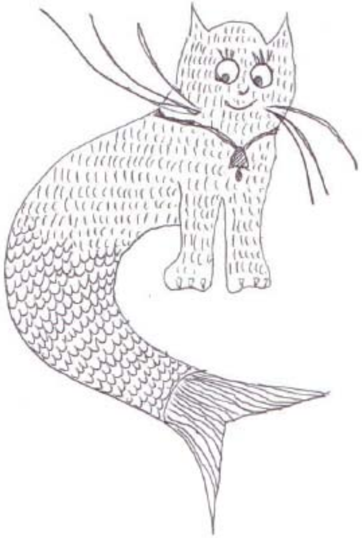
superfluid

$$n_n(T,n) = \frac{\hbar^2}{2\pi M k_B T}\int_0^\infty \frac{q^3 e^{\hbar\omega(q)/k_BT}}{(e^{\hbar\omega(q)/k_BT} - 1)^2}dq$$

Conclusions to these lectures:

In Bose-Fermi systems with direct repulsive interaction of bosons and fermions:

1. Fermions attract fermions which results in Cooper pairing
2. Excitons are bosons whose concentration may be controlled optically
3. Light-induced superconductivity may lead to the increase of critical temperature in conventional superconductors
4. Bosons attract bosons which results in formation of the roton minimum and suppression of BEC



Experiment needed!