On the Determination of the Optical Constants of Semiconductor Thin Films from Photometric Measurements.* Paul M. Grant†, Harvard University.

Theoretical studies have been carried out on the accuracy of derivation of the optical constants n and k of semiconductor thin films in the wavelength range 2000 Å to 6000 Å from measurements of the normal incidence transmissivity and reflectivity coefficients T and R. In order to determine the extent to which the errors in the derived optical constants depend upon experimental errors in R and T, the partial derivatives $\frac{\delta n}{\delta R}$, $\frac{\delta n}{\delta T}$, $\frac{\delta k}{\delta R}$, $\frac{\delta k}{\delta T}$, were calculated using appropriate theoretical equations and germanium optical constants obtained by a Kramers - Kronig analysis[1] of the reflectivity data of Donovan, Ashley, and Bennett[2]. The film thicknesses considered were from 50 Å to 500 Å. The results indicate that in the wavelength regions where $n \approx k$, the error in the derived optical constants becomes intolerably large for the usual experimental errors in R and T. Values of n and k calculated from actual measurements of R and T for epitaxial films of germanium deposited in vacuo on single crystal $CaF_2$ substrates are also reported and are shown to substantiate our theoretical conclusions.

* Research supported by the Office of Naval Research

† International Business Machines Corporation Fellow

1. H.R. Philipp, private communication.

2. T.M. Donovan, E.J. Ashley, and H.E. Bennett, J. Opt. Soc. Am. 53, 1403 (1963).

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of radions. Electron—sinks and protons—sources interact in the radional field with a force $f_e = e_1e_2/R^2 = J_t\eta^2\pi a_1^2\pi a_2^2/R^2 = J_t\beta^2 m_1 m_2/R^2$ dyn. $J_t$ is field pressure, $m$ mass, $a$ radius of the particle, $\eta$ numeric, equal to or multiple of 11.7, $\beta = 1$ cm²/g, $R$ distance between the particles, and $e$ elementary charge. Deviation of radional field from isotropy causes electrostatic field. Circular deviation from isotropy produces magnetic field. Multiple rebound of radions between elementary particles produces nuclear forces. For gravitational interaction see Ref. 1.

* PIA, CERG (USA), 44, 1945 Calvert St, Washington, D. C. 20009.
[1] A. J. Schneiderov, Bull. Am. Phys. Soc. 9, 400 (1965).

**KE14. Rotation of the Universe.** HENRY GREBER.—The here-suggested hypothesis is to account for the red shift of light from extragalactic bodies, the gravitational equilibrium of the universe, Olber's paradox, and Mach's principle. It is based on the assumption that galactic clusters, like electrons in an atom, rotate around the center of the universe, so that the centrifugal force acting upon them balances the gravitational attraction of their center of gravity. Accordingly, the red shift can be explained as a relativistic effect of light emitted from rotating sources. The support of this hypothesis rests on that, in order not to exceed the velocity of light $c$, a photon can leave a body rotating with the peripheral velocity $v$, only with the velocity $w = (c^2 - v^2)^{\frac{1}{2}}$. For the energy of the photon to be constant, according to $E = fh = (w/1)h$, with $f$ the frequency, 1 the wavelength, and $h$ being Planck's constant, 1 must be proportional to $w$, and $1'/1 = c/(c^2 - v^2)^{\frac{1}{2}} \approx 1 \mp v/2c$. From which the red shift is $d1/1 = v/c$. Since, by definition, $(d1/1)c = HR$, $H$ being Hubble's constant, and $R$ being the distance of the rotating body from the center of the universe. For $R = R_{max}$ ($R_{max}$ is the radius of the universe), $d1/1 = 1$ and $v$, can be maximally equal to $c$. Hecne $H = c/R_{max} = 2\times10^{10}/10^{27} = 2\times10^{-17}$, close to the observational value $1.88\times10^{-17}$.

THURSDAY AFTERNOON, 29 APRIL 1965 EAST BUILDING NATIONAL BUREAU OF STANDARDS AT 1:30

(W. W. SCANLON presiding)

## Semiconductors II

**KF1. Helicon-Wave Propagation in $n$-Type InAs at Microwave Frequencies.** J. K. FURDYNA, *National Magnet Laboratory* MIT.*—Helicon-wave propagation was investigated in $n$-type InAs at 35 Gc/sec, in the temperature range from 78° to 300°K in magnetic fields up to 100 kG. A microwave analog of the Rayleigh refractometer was used in the measurements. This arrangement is in many ways superior to the Fabry–Perot dimensional resonance technque used in previous helicon experiments in this frequency range, particularly in the presence of considerable losses. Moreover, this approach permits a quantitative study of the helicon-damping processes. Interference patterns consisting of as many as 10 full oscillations between 40 and 100 kG were observed at temperatures as high as 300°K, allowing a precise determination of the electron concentration. In addition, the quantity $e/(m^*\langle\tau^{-1}\rangle)$ was obtained from the envelope of the interference pattern. The latter quantity is close to the known electron mobility, but shows a slight dependence on the magnetic field. This variation arises possibly due to the proximity of the quantum limit, which may render the semiclassical model used in the present analysis not fully satisfactory at the highest fields.

* Work supported by the U. S. Air Force Office of Scientific Research.

**KF2. Method for Observing Cyclotron Resonance at Millimeter Wavelengths.*** R. KAPLAN, *U. S. Naval Research Laboratory.*—A method is described for observing cyclotron resonance in materials of which the extrinsic photoconductivity depends on carrier energy. The method is derived from a technique described earlier.[1] Steady illumination of a sample produces carriers, either holes or electrons, by ionization of shallow impurity levels. Microwave radiation absorbed by the carriers raises their energy slightly, thus changing the sample's conductivity. Cyclotron-resonance spectra are observed by monitoring the conductivity as an applied magnetic field is scanned through an appropriate range. The method is particularly useful for measurements at the shorter millimeter wavelengths for the following reasons: high sensitivity may be achieved, absorption by holes or electrons may be observed separately, and a minimum of microwave circuitry is required. The mechanisms responsible for the variation of sample conductivity with carrier energy are considered, and experiments performed at wavelengths of 2 and 4 mm are described.

* Work performed under Project Defender, sponsored by the Advanced Research Projects Agency.
[1] H. J. Zeiger, C. J. Rauch, and M. E. Behrndt, Phys. Chem. Solids 8, 496 (1959).

**KF3. Oscillatory Faraday Rotation of the Indirect Transition in Germanium.** JOHN HALPERN,* *Lincoln Laboratory† MIT.*—The oscillatory Faraday rotation of the indirect transition in germanium has been observed on transmission. The experiments were carried out at magnetic fields of up to 103 kG and at helium temperatures on a heat-sunk sample. From the position of the indirect energy gap, the temperature was determined as 8°K. The sample was intrinsic, with a carrier concentration $n < 10^{13}$/cm³. The data were taken with the direction of propagation (and the magnetic field) perpendicular to a ⟨110⟩ face. Comparison of the Faraday rotation with the corresponding transmission as a function of wavelength shows that there are oscillatory curves corresponding to both the exciton absorption and to the Landau steps. The entire indirect transition rotation is superposed on a very large dispersive tail that probably arises from the $\Gamma_{25'} \rightarrow \Gamma_{2'}$ direct-energy-gap transition. At the temperatures and magnetic fields in question, the oscillatory effects are of the order of 2% of the total rotation.

* Visiting scientist at the National Magnet Laboratory MIT, which is operated with support from the U. S. Air Force Office of Scientific Research.
† Operated with support from the U. S. Air Force.

**KF4. Multiple Reflection Effects in Faraday Rotation in Semiconductors.** E. D. PALIK, J. R. STEVENSON,* B. W. HENVIS, *U. S. Naval Research Laboratory,* B. DONAVAN, AND J. WEBSTER, *Westfield College, University of London.*—We have measured the free-carrier Faraday rotation in an $n$-type PbS epitaxial film at room temperature in order to study the effects of multiple reflections on rotation. The sample was 4.6 $\mu$ thick and contained $1.8\times10^{18}$ carriers/cm³. Measurements were made in the spectral region 3–30 $\mu$ with magnetic fields as high as 132 kOe. Just as the transmission of the film showed well-defined interference fringes as a function of wavelength owing to multiple reflections, the Faraday rotation also showed oscillations that were in step with the transmission fringes. Extensive calculations based on the work of Donovan and Medcalf[1] have been carried out to fit the observed rotation and to determine the dependence of the rotation on the various parameters such as carrier concentration, carrier relaxation time, sample thickness, and magnetic field.

† Permanent address: Georgia Institute of Technology.
[1] B. Donovan and T. Medcalf, Brit. J. Appl. Phys. 15, 1139 (1964).

**KF5. Determination of the Optical Constants of Semiconductor Thin Films from Photometric Measurements.*** Paul M. Grant,† *Harvard University.*—Theoretical studies have been carried out on the accuracy of derivation of the optical constants $n$ and $k$ of semiconductor thin films in the wavelength range 2000–6000 Å from measurements of the normal incidence transmissivity and reflectivity coefficients $T$ and $R$. In order to determine the extent to which the errors in the derived optical constants depend upon experimental errors in $R$ and $T$, the partial derivatives $\delta n/\delta R$, $\delta n/\delta T$, $\delta k/\delta R$, $\delta k/\delta T$ were calculated, using appropriate theoretical equations and germanium optical constants obtained by a Kramers'–Kronig analysis[1] of the reflectivity data of Donovan, Ashley, and Bennett.[2] The film thicknesses considered were from 50 to 500 Å. The results indicate that, in the wavelength regions where $n\approx k$, the error in the derived optical constants becomes intolerably large for the usual experimental errors in $R$ and $T$. Values of $n$ and $k$ calculated from actual measurements of $R$ and $T$ for epitaxial films of germanium deposited *in vacuo* on single-crystal $CaF_2$ substrates are also reported and are shown to substantiate our theoretical conclusions.

* Research supported by the U. S. Office of Naval Research.
† International Business Machines Corporation Fellow.
[1] H. R. Philipp, private communication.
[2] T. M. Donovan, E. J. Ashley, and H. E. Bennett, J. Opt. Soc. Am. 53, 1403 (1963).

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**KF6. Electrical Properties of Clean Surfaces of Degenerate Germanium.** J. E. Alberghini and R. M. Broudy, *United Aircraft Research Laboratories.*—Clean surfaces of highly doped $n$-type germanium of carrier concentration $n\cong 2\times10^{19}$ cm$^{-3}$ have been prepared by cleavage in liquid nitrogen and measurements of surface conductance have been made therein; in this, ambient surfaces remain clean for many hours. The germanium is cut and oriented in the major crystal coordinate system ⟨111⟩, ⟨112⟩, ⟨110⟩, the cleavage direction being ⟨110⟩ and the cleavage face being the {111} cleavage plane; cleaving in this coordinate system gives smoother, more-reproducible faces than the usual random orientation in the {111} cleavage plane. Surface conductance has been measured between two $p$–$n$ junctions placed along the cleavage front in a manner analogous to Handler.[1] The $p$–$n$–$p$ structure provides a high resistance to the bulk, which is caused by the low *forward* conductance at liquid-nitrogen temperature of properly prepared nontunneling junctions below 0.7 V bias. Measurements on nondegenerate germanium agree roughly with the results of others, whereas preliminary measurements on degenerate samples indicate either zero or negligible surface conductance.

[1] P. Handler, Appl. Phys. Letters 3, 96 (1963).

**KF7. Hall Measurements on Inverted Surfaces of $P$-Type Silicon.** Alan B. Fowler, *IBM Watson Research Center.*—Hall measurements were made on silicon surfaces as a function of the electric field applied across thermally grown oxides. Substrates of $p$-type silicon with resistivities from 1 to 100 Ω·cm were studied. The mobility was found to increase as the inversion was increased up to a maximum value at a surface-charge density of the order of $10^{10}$ electrons/cm$^2$. At higher fields, the mobility decreased. The maximum value of mobility was found to vary significantly as a function of substrate doping and heat treatment. The maximum value of Hall mobility of 800 cm$^2$/V·sec was observed on a 100 Ω·cm substrate. The maximum value for 1 Ω·cm was 400 cm$^2$/V·sec. Heat treatment at 350°C to remove surface traps also decreased the mobility.

**KF8. Magnetoresistance of $P$-doped Germanium in the Hop Conduction Range.*** W. W. Lee (introduced by R. J. Sladek) and R. J. Sladek, *Purdue University.*—Measurements of magnetoresistance have been made at liquid-helium temperatures and at 77°K on phosphorus-doped germanium samples having room-temperature carrier concentrations between $7\times10^{15}$ and $3.5\times10^{16}$ cm$^{-3}$. Field strengths up to 25 kG were employed. It is found that at low temperatures $\rho/\rho_0$ exhibits a crystalline anisotropy and field dependence that are different than those for the conduction band, in general agreement with previous results on Sb-doped Ge (Ref. 1). The explanation[1,2] of the latter, and presumably of the present results also, was that the magnetic field influences the donor wavefunctions and thereby, the jumping of electrons between donors, which is the important conduction mechanism at low temperatures for the samples in question. The magnitude of $\rho/\rho_0$ for P-doped Ge is smaller than that for Sb-doped Ge of comparable donor concentration, as would be expected, owing to the smaller Bohr radius around the P atom. The dependence of $\rho/\rho_0$ on temperature was also investigated. Some possible explanations of these results are given.

* Work supported by the U. S. Army Research Office (Durham).
[1] R. J. Sladek and R. W. Keyes, Phys. Rev. 122, 437 (1961).
[2] N. Miloshiba, Phys. Rev. 127, 1962 (1962).

**KF9. Use of MOS Capacitors in Determining Properties of Surface States at the Si–$SiO_2$ Interface.*** Jean F. Delord, Dennis G. Hoffman, *Reed College,* and Gene Stringer, *University of Oregon.*—The dynamic capacitance–voltage characteristics of metal–oxide–silicon (MOS) capacitors were studied as a function of silicon-crystal orientation, oxide thickness, impurity concentration, bias voltage, and ambient atmosphere. Measurements were made with a Tektronix capacitance-curve tracer using a fixed 80-kc/sec sampling frequency-superimposed on an externally variable dc bias. This allows direct capacitance–voltage and capacitance–time displays on the oscilloscope. Typical values obtained for the surface-charge density (electrons/cm$^2$) were $4\times10^{11}$, $6\times10^{11}$, $1.2\times10^{12}$ for the (100), (110), and (111) orientations of the silicon planes, respectively. With the application of a step bias–voltage, the capacitance approached its equilibrium value at a rate dependent upon the oxide thickness, silicon orientation, and the magnitude of the applied bias; reaching equilibrium in the order of 10ths of seconds. With water vapor present in the ambient, the repeated cycling of the bias voltage decreased the time to reach equilibrium by up to a factor of 5. An explanation of the experimental results is presented based upon the assumption that the interface surface states are of the Read–Shockley type.

* Experimental measurements were made at Tektronix Inc., Beaverton.

**KF10. Scattering Amplitudes for 80-keV Electrons in Diamond, Silicon, and Gray Tin.** H. A. Fowler, *National Bureau of Standards.*—Elastic-scattering amplitudes for small-angle scattering of electrons at 80 keV have been measured by electron-diffraction techniques, using transmission through single crystals of diamond, silicon, and gray tin. One method utilizes Kikuchi-line widths, another, thickness-extinction contours. Both differential and absolute scattering amplitudes exhibit quantitative agreement with scattering calculations by Ibers[1] and Zeitler and Olsen.[2] Variation with $Z$ is as expected from

# Determination of the Optical Constants of Semiconductor Thin Films from Photometric Measurements

## Introduction.

At the Gordon McKay Applied Physics Laboratory of Harvard, we are presently engaged in research into the optical properties of thin epitaxial semiconductor films. One of the initial problems chosen for study was the calculation of the optical constants of germanium in the wavelength range 2000 Å to 6000 Å through the measurement of the normal incidence reflectivity and transmissivity coefficients of epitaxial films. During the course of this work it was noticed that there repeatedly occured a wavelength region in which very small errors in R and T gave large errors in n and k. Therefore, a detailed investigation of the linear, first order dependence of n and k upon R and T and the film thickness was made using appropriate theoretical equations and germanium optical constants obtained by a Kramers-Kronig analysis of the reflectivity data of Donovan, Ashley and Bennett.

## Slide 1: Theoretical Equations

a. Equations (1) and (2) of this slide represent the functional dependence of R and T on the optical constants n and k and the parameters λ, wavelength, and a, film thickness. [redacted] We will assume that (1) and (2) contain all effects due to interference and a finite substrate. By using appropriate numerical procedures, [redacted] such as a Newton-Raphson iteration, (1) and (2) may be inverted to yield n and k. However, in doing so, large errors in n and k may appear due to small errors in R and T, [redacted] but the discussion to follow will give [redacted] results independent of any numerical techniques used.

b. Equations (3) through (6) give the first order dependence of n, k upon R, T. They also give the effect of experimental errors in R, T upon n, k. We note that the implicit derivatives, or error derivatives, depend on the explicit derivatives through the jacobian. Although physics requires smoothness in the behavior of the explicit derivatives, a vanishing jacobian could cause singularities in the implicit derivatives, in which case the inversion is not valid.

In passing, we also mention that in the Newton-Raphson iteration, a vanishing jacobian cause the numerical procedure to diverge.

An inevitable question is how does this method compare with the Kramers-Kronig analysis. Although a detailed examination of the latter, analogous to ours for the R-T method, has not been published, preliminary indications are that [redacted] dispersion analyses are superior to film determinations even when the film quality is high.

c. The theory outlined in this talk in addition to experimental data from epitaxial germanium films will be presented shortly in a thesis and technical report at Harvard.

Slide 2: $\left|\frac{\partial n}{\partial R}\right|$ vs. $\lambda$

a. It turns out that $\frac{\partial n}{\partial R}$ is the most sensitive and critical error derivative. This slide gives the wavelength dependence of its magnitude for several film thicknesses as found by straightforward calculation from the previous equations.

b. We see that $\left|\frac{\partial n}{\partial R}\right|$ does indeed have a singularity at at least one given wavelength which ~~that~~ changes somewhat with film thickness. We further note that for $\left|\frac{\partial n}{\partial R}\right|$ greater than 20, a 2.5 % absolute error in R gives an error of 1/2 of an optical constant in the derived results. We will arbitrarily take this value of 1/2 as our criterion for an excessive error.

Slide 3: Root-Locus Plot.

a. This slide gives the locus of all roots, n, k, for a 150Å film at wavelength 3000Å, which satisfy the R and T equation separately, the intersections of course giving the simultaneous solutions. We have purposely chosen the wavelength to be near the point of singularity in $\frac{\partial n}{\partial R}$ as shown in the previous slide. We note in passing that there are several possible solutions from which we must choose the physically meaningful one which here turns out to be this one.

b. The geometric interpretation of the jacobian in terms of these root-locus ~~curves~~ plots is that it is proportional to the cross product magnitude of the normal derivatives, hence slopes, of these curves at their point of intersection. Therefore, when the jacobian vanishes, the slopes are equal which we see to be the case here. Note that small changes in the R and T curves due to experimental errors will cause either no root at all or quite large errors in the optical constants, particularly n.

Point out meaning of n and k axes and label R and T curves. These curves show independence of root-getting methods.

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Slide 4: $\left|\frac{\partial n}{\partial R}\right|$ : $\lambda$ Range vs. Thickness

a. Recall that we had mentioned that for $\frac{\partial n}{\partial R}$ greater than 20, a 2.5% error in $R$ gave an error of ½ in $n$. This slide shows the wavelength range, indicated by bars, over which $\frac{\partial n}{\partial R}$ exceeds 20 as a function of film thickness. The "X" represents the wavelength at which $\frac{\partial n}{\partial R}$ goes to infinity.

b. We see that there is definitely a thickness range over which the extent of excessive values of $\frac{\partial n}{\partial R}$ is limited to smaller wavelength intervals and in planning an experiment one should choose a film thickness which lands in this range. We also see that, for germanium anyway, there is no way in which two films can be chosen which do not have overlapping regions of excessive $\frac{\partial n}{\partial R}$.

c. When one neglects interference and internal reflections altogether, one can show that singularities occur in $\frac{\partial n}{\partial R}$ regardless of film thickness whenever $n = \sqrt{k^2+1}$. For germanium, the wavelength at which this occurs is indicated by the dagger to be about 3000 Å. This is quite close to the point where most of the singularities occur when interference is not neglected.

## Slide 5: Optical Constants of 250Å Ge Film

a. This slide shows the optical constants, given by the broken line, calculated from R and T measurements on a 250Å epitaxial germanium film. The solid line gives the results for bulk germanium via a Kramers - Kronig analysis of reflectivity data.

b. We see that there is a region in which no roots were found which corresponds roughly to our predicted region of excessive $\frac{\partial n}{\partial R}$. However, in the region where roots are obtained, the agreement with bulk values is much better than any previously reported thin film data.

Slide 6: Table of Other Semiconductor Films

a. We would like to point out that the problem of excessive first order implicit derivatives exists for materials other than germanium when the R-T method of calculating optical constants is used. This table was compiled from published Kramers-Kronig data and gives the wavelength at which $n = \sqrt{k^2+1}$ for various semiconductors. We see ~~[illegible]~~ invariably that there is some wavelength at which this always occurs, a fact which we could deduce from the general similarity of semiconductor band structures. In conclusion we would like to remark that whenever the R-T photometric method is used to determine optical constants from film data, a theoretical investigation, if possible, should be made to determine the wavelength range of high sensitivity to experimental error. ~~The theory outlined in this talk [illegible] discussed in addition to experimental data from epitaxial germanium films will be presented shortly in a thesis and technical report [illegible] at Harvard.~~

$$R(n,k) - R = 0$$

$$T(n,k) - T = 0$$

$$R = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$$

$$T = (1-R)^2 \, e^{-\frac{4\pi}{\lambda} k a}$$

$$k = \frac{\lambda}{4\pi a} \ln \frac{(1-R)^2}{T}$$

$$n = \frac{1+R}{1-R} + \left\{ \frac{4R}{(1-R)^2} - \frac{\lambda^2}{16\pi^2 a^2} \left[ \ln \frac{(1-R)^2}{T} \right]^2 \right\}^{1/2}$$

$$\text{DISCRIMINANT} = n^2 - k^2 - 1$$

$$R = R(n, k; d; a) \tag{1}$$

$$T = T(n, k; d; a) \tag{2}$$

$$dn = \frac{\partial n}{\partial R}\, dR + \frac{\partial n}{\partial T}\, dT \tag{3}$$

$$dk = \frac{\partial k}{\partial R}\, dR + \frac{\partial k}{\partial T}\, dT \tag{4}$$

where $$\frac{\partial n}{\partial R} = \frac{\partial T}{\partial k} \Big/ J \ , \text{ etc.} \tag{5}$$

$$J = \frac{\partial R}{\partial n}\frac{\partial T}{\partial k} - \frac{\partial R}{\partial k}\frac{\partial T}{\partial n} \tag{6}$$

$$R = R_{FA} + \frac{T_{FS}^2 \, R_{AS}}{1 - R_{AS} \, R_{FS}} \tag{1}$$

$$T = \frac{T_{FS} \, (1 - R_{AS})}{1 - R_{AS} \, R_{FS}} \tag{2}$$

where: $$R_{AS} = (1-n_s)^2 / (1+n_s)^2 \tag{3}$$

$$R_{FA} = R_{FA}^N \left\{ \left( e^{\alpha a/2} - (R_{FS}^N / R_{FA}^N)^{1/2} e^{-\alpha a/2} \right)^2 + 4 (R_{FS}^N / R_{FA}^N)^{1/2} \sin^2 \left( \phi + (\psi_{FS} - \psi_{FA})/2 \right) \right\} / D \tag{4}$$

$$R_{FS} = R_{FS}^N \left\{ \left( e^{\alpha a/2} - (R_{FA}^N / R_{FS}^N)^{1/2} e^{-\alpha a/2} \right)^2 + 4 (R_{FA}^N / R_{FS}^N)^{1/2} \sin^2 \left( \phi + (\psi_{FA} - \psi_{FS})/2 \right) \right\} / D \tag{5}$$

$$T_{FS} = \left\{ \frac{n_s}{[(1+n)^2 + k^2][(n+n_s)^2 + k^2]} \right\} \left\{ 16 \, (n^2 + k^2) / D \right\} \tag{6}$$

$$D = \left( e^{\alpha a/2} - (R_{FA}^N R_{FS}^N)^{1/2} e^{-\alpha a/2} \right)^2 + 4 (R_{FA}^N R_{FS}^N)^{1/2} \sin^2 \left( \phi + (\psi_{FA} + \psi_{FS})/2 \right) \tag{7}$$

$$R_{FA}^N = \frac{(1-n)^2 + k^2}{(1+n)^2 + k^2} \quad ; \quad R_{FS}^N = \frac{(n_s - n)^2 + k^2}{(n_s + n)^2 + k^2} \tag{8}$$

$$\psi_{FA} = \tan^{-1} \left\{ \frac{2k}{n^2 + k^2 - 1} \right\} \quad ; \quad \psi_{FS} = \tan^{-1} \left\{ \frac{2 n_s k}{n^2 + k^2 - n_s^2} \right\} \tag{9}$$

$$\alpha = 4\pi k / \lambda \quad ; \quad \phi = 2\pi n a / \lambda \tag{10}$$

$$dn = \frac{\partial n}{\partial R} dR + \frac{\partial n}{\partial T} dT \quad ; \quad dk = \frac{\partial k}{\partial R} dR + \frac{\partial k}{\partial T} dT \tag{11}$$

where: $$\frac{\partial n}{\partial R} = \frac{\partial T}{\partial k} / J \text{ , etc.} \tag{12}$$

$$J = \frac{\partial R}{\partial n} \frac{\partial T}{\partial k} - \frac{\partial R}{\partial k} \frac{\partial T}{\partial n} \tag{13}$$