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METHOD AND MEANS FOR HYPERGEOMETRIC FUNCTION CALCULATION ON AN ARRAY PROCESSOR

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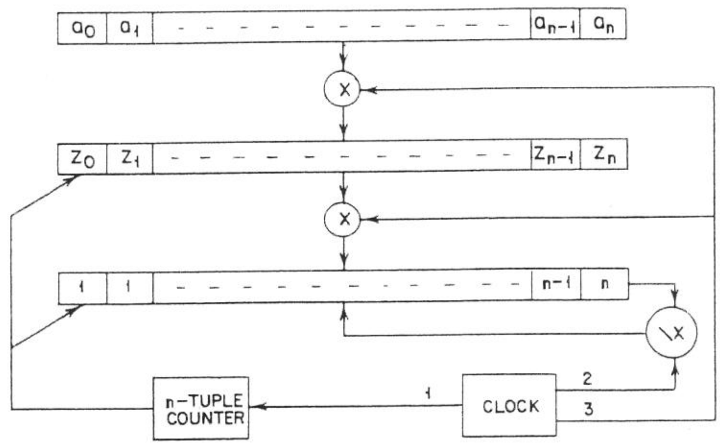

TIMING: 1 BEFORE 2 BEFORE 3

This article describes a method for efficient calculation in the hardware of an array processor, of those hypergeometric functions which are defined by successive differentiation of a seed function. Examples are the Legendre functions, defined by:

$$P_{\ell m}(z) = (1-z^2)^{m/2} \frac{d^m P_\ell(z)}{dz^m} .$$

where $P_\ell(z)$ is the ℓ-th Legendre polynomial; the associated Laguerre functions, defined by:

$$L_r^s(\rho) = \frac{d^s}{d\rho^s} L_r(\rho),$$

where $L_r(\rho)$ is the 4-th Laguerre polynomial; and the spherical Bessel functions, defined by:

$$j_n(z) = z^n \left(- \frac{1}{z} \frac{d}{dz}\right)^n \frac{\sin z}{z} .$$

The attribute which permits the utilization of an array processor in the calculation of such functions resides in the fact that the seed function can be usually represented as a polynomial whose general term is simply:

$$a_n z^n$$

with general derivative

$$\frac{d^m}{dz^m} a_n z^n = a_n \left(\frac{n!}{(n-m)!} \right) z^{n-m} .$$

Thus, if in the calculation of the seed function polynomial, either by recursion as in the case of Legendre and Laguerre polynomials, or by economization as for spherical Bessel functions, one forms an appropriate array of coefficients and exponents, the calculation of the required hypergeometric function becomes a trivial shift and multiply operation of this array in hardware capable of parallel array operations. Often the seed function itself can be calculated via such array manipulations, as, for example, when we define the Legendre polynomials by Rodriquez' formula:

$$P_\ell(z) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dz^\ell} (z^2-1)^\ell .$$

The operating steps of the method may be expressed in the following APL sequence.

```
     ∇ALF[⎕]∇
     ∇ PLM←LM ALF Z;PL;PL2;EL;EL2;LL;FEL;I
[1]  ASSOCIATED LEGENDRE FUNCTION
[2]  FUNCTIONAL FORM:  P(L,M,Z)←(L,M) ALF Z
[3]  ALGORITHM USES ORDINARY LEGENDRE POLYNOMIAL RECURSION EQUATION
[4]  FOLLOWED BY DERIVATIVE DEFINITION OF ASSOCIATED FUNCTION
[5]  →(PLM←LM[1]=0)/0
[6]  →(1=LM[1]=0)/0
[7]  PL←PL2←,1
[8]  EL←1+EL2←,1
[9]  LL←2
[10] S1:PL←(÷LL)x((¯1+2xLL)xPL2←PL),PL2x1-LL
[11] EL←(1+EL2←EL),,EL2
[12] →(LM[1]≥LL+1)/S1
[13] PL←((EL←FEL,FEL+2x⍳(LM[1]-FEL←L/EL)·2)∘.=,EL)+.xPL
[14] PLM←((1-Z*2)*0,5xLM[2](((ρ,Z),ρI)ρPL[I]x(!EL[I]+LM[2])
     ÷!EL[I])x(,Z)°.*EL[I←EL≥0)/⍳ρEL←EL-LM[2]]
[15] →0
[16] S2:→(0=LM[2])/3
[17] PLM←(1-Z*2)*0.5
[18] →0
[19] S3:PLM←Z
    ∇
     ∇NALF[⎕]∇
    ∇ TLM←LM NALF Z
[1]   NORMALIZED ASSOCIATED LEGENDRE FUNCTION
[2]  TLM←(LM ALF Z)x(0.5x(1+2xLM[1])x(!-/LM)÷!-LM)*
     0.5
    ∇
```

In function ALF, lines 10-12 compute the coefficients and exponents of the seed Legendre polynomial by recursion. Lines 13-14 compute the required value of the associated Legendre function by the array manipulation algorithm discussed in this article.

The figure diagrammatically depicts the parallel processor architecture necessary for calculating hypergeometric function through application of Eqs. (4) and (5). The an, Zn, and n arrays are assumed to be previously loaded as is also the m-counter. The result will be accumulated in the n array. The n array is provided with a cumulative multiplier, that is, the product developed in the ith element is the product of i, i-1, i-2, ..., i-m + 1. The numbers 1, 2, and 3 emanating from the clock denote the timing sequence.