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Fundamental limitations on plasma fusion systems not in thermodynamic equilibrium

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Analytical Fokker–Planck calculations are used to accurately determine the minimum power that must be recycled in order to maintain a plasma out of thermodynamic equilibrium despite collisions. For virtually all possible types of fusion reactors in which the major particle species are significantly non-Maxwellian or are at radically different mean energies, this minimum recirculating power is substantially larger than the fusion power. Barring the discovery of methods for recycling the power at exceedingly high efficiencies, grossly nonequilibrium reactors will not be able to produce net power. [S1070-664X(97)01404-3]

I. INTRODUCTION

One of the most important challenges in modern physics is to identify the best approach to clean and efficient fusion power generation. Advanced aneutronic fuels such as ^{3}He–^{3}He, p-^{11}B, and p-^{6}Li would produce considerably less neutron radiation and radioactive by-products than more conventional fusion fuels like deuterium–tritium (D–T) and deuterium–deuterium (D–D), and furthermore they might permit high-efficiency direct electric conversion of the fusion energy instead of low-efficiency thermal conversion. Unfortunately, plasma systems which are essentially in thermodynamic equilibrium cannot break even against radiation losses with these aneutronic fuels;[1] for this reason, it has been suggested that plasma fusion systems which are substantially out of thermodynamic equilibrium should be considered.[2] As a further incentive for the study of nonequilibrium fusion plasmas, the somewhat more conventional fuel D–^{3}He could be made cleaner and more attractive if it were possible to suppress undesirable D–D side reactions more than can be done in an equilibrium D–^{3}He plasma.[1]

This paper will resolve the question of whether highly nonequilibrium plasma systems would be useful for fusion purposes, especially with regard to advanced-fuel fusion. Rather than limit the analysis to a particular type of nonequilibrium fusion reactor design, it would be wise to make this study as generally applicable as possible. Accordingly, a minimum of assumptions will be made with regard to the plasma geometry, reactor confinement system, type of fuel, and other key parameters. Those assumptions which are made are as follows:

(i) Losses other than bremsstrahlung radiation and the power required to keep the plasma out of thermodynamic equilibrium are ignored, so this analysis sets an optimistic bound on the performance of nonequilibrium fusion reactors.

(ii) Energy transfer from the fuel ions to the electrons is the only energy source available to the electrons; the electrons cannot acquire energy from external heating systems, fusion products, or other sources. This assumption sets a lower limit on the electron temperature and bremsstrahlung losses, in agreement with the stated goal of finding an optimistic bound on the performance of the fusion systems.

(iii) Likewise, it is optimistically assumed that the entire fusion reaction output power can be utilized. Conversion efficiency limitations are ignored, and power losses are directly compared with the gross fusion power P_{fus}.

(iv) In comparing collisional scattering effects, fusion, and bremsstrahlung radiation with each other, the density, spatial density profiles, and plasma volume do not matter, since all of these phenomena are two-body effects and thus are proportional to $\int d^3\mathbf{x}[n(\mathbf{x})]^2$ (neglecting the weak density dependence of the Coulomb logarithm $\ln \Lambda$), in which $n(\mathbf{x})$ is the particle density as a function of position.

(v) The regions of the plasma which have values of $\int d^3\mathbf{x}[n(\mathbf{x})])^2$ large enough to be of interest are approximately isotropic. Otherwise they would be subject to counterstreaming,[3] Weibel,[4] and other instabilities.

(vi) Although instabilities can prove to be a serious concern even in essentially isotropic nonthermal plasmas, they will be optimistically ignored here.

(vii) Spatial variations of particle energies may be neglected in regions of significant $\int d^3\mathbf{x}[n(\mathbf{x})]^2$.

(viii) The plasma is quasineutral and optically thin to bremsstrahlung.

(ix) In the ion energy ranges of interest, the functional dependence of the fusion reactivity $\langle \sigma v\rangle_{\text{fus}}$ on the mean ion energy $\langle E_i\rangle$ is approximately independent of the ion velocity distributions' shapes if the distributions are isotropic and the ion species have the same mean energy, as shown explicitly in Ref. 5.

(x) The functional dependence of the bremsstrahlung radiation power on the mean electron energy $\langle E_e\rangle$ is approximately independent of the electron velocity distribution shape in the energy range of interest.

As demonstrated in Ref. 5, systems which violate the

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above assumptions would not offer advantages substantial enough to be of particular value, even if such systems could be constructed.

All quantities are in cgs units, with energies and temperatures both measured in ergs, unless otherwise stated. For ready comparison with equilibrium plasmas, the ''temperature'' T of a non-Maxwellian distribution with a mean particle energy $\langle E \rangle$ is defined as $T \equiv 2\langle E \rangle/3$.

In Ref. 1, it was shown that plasma systems which are intended to operate far from thermodynamic equilibrium and yet which have no specific provisions for maintaining such a state will very rapidly relax to equilibrium. Methods of passively maintaining nonequilibrium distributions have also been shown to be inadequate.[6] Therefore the present discussion will focus on systems which maintain nonequilibrium plasmas by active but otherwise arbitrary means. Such active maintenance of nonequilibrium plasmas will entail certain minimum power requirements and limitations.

In Sec. II we will examine the limitations that affect plasma systems which attempt to maintain substantially non-Maxwellian velocity distributions for the electrons or the fuel ions. In Sec. III we will then present the fundamental limitations that pertain to plasma systems in which two or more of the major particle species are at radically different mean energies. Using the results of Secs. II and III, in Sec. IV we will discuss the implications for controlled fusion.

II. NON-MAXWELLIAN VELOCITY DISTRIBUTIONS

One way in which a plasma can deviate from thermodynamic equilibrium is to have non-Maxwellian particle velocity distributions. While no fusion system has perfectly Maxwellian distributions, the systems which will be considered here are of interest because they deviate from the Maxwellian equilibrium in a much more marked fashion than is usual.

A. Preliminary estimates

Before performing a rigorous derivation of the minimum power requirements needed to maintain non-Maxwellian velocity distributions, it is useful to make preliminary estimates of these power requirements as a means of gaining physical insight into the problem. Estimates will be made for two different types of velocity distribution functions. For simplicity only one particle species will be considered.

First consider an isotropic beam-like velocity distribution in which the particles are centered around a mean speed v_0 with some ''thermal'' spread $v_t \ll v_0$ on each side of the mean speed. Due to collisions, a certain number (actually a certain density) of the particles $n_{\rm fast}$ will gain an amount of energy $\Delta E_{\rm fast}$ on a timescale of $\tau_{\rm fast}$. If the width of the distribution is to be kept from spreading beyond the allowed v_t, then one must extract a power density $P_{\rm recirc}$ from the particles which have become too fast and give it to particles which have become too slow. This quantity $P_{\rm recirc}$ is defined as

$$P_{\rm recirc} = \frac{n_{\rm fast}\Delta E_{\rm fast}}{\tau_{\rm fast}}. \tag{1}$$

The parallel velocity-space diffusion coefficient for a particle with velocity $v_{\rm test}$ in the presence of isotropic, monoenergetic field particles of the same species with speed v_0 is[7]

$$D_\parallel \approx \frac{\sqrt{\pi}}{3\sqrt{6}} \left(\frac{v_0}{v_{\rm test}}\right)^3 \frac{v_0^2}{\tau_{\rm col}}, \tag{2}$$

where the usual definition of the collision time $\tau_{\rm col}$ (Ref. 8) has been used with $\langle E \rangle = (3/2)T \approx m v_0^2/2$:

$$\tau_{\rm col} \equiv \frac{\sqrt{m}\langle E \rangle^{3/2}}{2\sqrt{3\pi}(Ze)^4 n \ln \Lambda}. \tag{3}$$

The time for a typical test particle to be collisionally upscattered from the velocity v_0 to the maximum allowed velocity $v_{\rm fast} \equiv v_0 + v_t$ may be estimated as

$$\tau_{\rm fast} \approx \frac{v_t^2}{D_\parallel} \approx \frac{3\sqrt{6}}{\sqrt{\pi}} \left(\frac{v_t}{v_0}\right)^2 \tau_{\rm col}, \tag{4}$$

where only the largest term has been retained.

By likewise keeping only the largest term of $\Delta E_{\rm fast}$ and using $\langle E \rangle \approx m v_0^2/2$, one finds the energy upscattering to be

$$\Delta E_{\rm fast} = \frac{1}{2} m (v_{\rm fast}^2 - v_0^2) \approx 2 \frac{v_t}{v_0} \langle E \rangle. \tag{5}$$

The final necessary assumption is that approximately half of the particles will be upscattered in energy and half will be downscattered, so $n_{\rm fast} \approx n/2$. By putting all of this information together, the recirculating power required to hold the proper distribution shape despite like-particle collisions is found to be

$$P_{\rm recirc} \approx \frac{\sqrt{\pi}}{3\sqrt{6}} \frac{v_0}{v_t} \frac{n\langle E \rangle}{\tau_{\rm col}} \approx 0.24 \frac{v_0}{v_t} \frac{n\langle E \rangle}{\tau_{\rm col}}. \tag{6}$$

The general form of this result will be confirmed by the more rigorous derivation.

The second case for which the minimum recirculating power will be estimated concerns velocity distributions which are nearly Maxwellian except that essentially all of the very slow particles in the distribution are depleted. This situation would be especially desirable for the electron distribution in advanced-fuel fusion plasmas, so that far fewer than the purely Maxwellian number of electrons would have speeds slower than the ions. Because ion–electron energy transfer is mediated by those slow electrons,[6] a large reduction in the electron temperature and radiation losses would result, and the power balance for the advanced fuels would be considerably improved.

Consider an electron distribution which looks superficially like a normal Maxwellian with a characteristic thermal velocity $v_{tf} \equiv \sqrt{2T_{0f}/m_e}$ but has no particles at speeds below some velocity v_0, which is chosen such that it is comparable to (actually somewhat greater than) the ion thermal velocity and obeys the relation $v_0 \ll v_{tf}$. Electron distributions which differ substantially from this while still keeping the slow electrons depleted will deviate further from the Maxwellian equilibrium state and hence be harder to maintain.

The recirculating power which must be continually extracted from the tail of the electron distribution and given to the slow electrons to boost their energies and maintain the ''hole'' in the center of the velocity distribution is

$$P_{\rm recirc}=\frac{n_{\rm slow}\Delta E_{\rm slow}}{t_E^{ee}}, \tag{7}$$

where $n_{\rm slow}$ is the density of slow electrons that must continually be acted upon, $\Delta E_{\rm slow}$ is the energy that must be given to each of them, and t_E^{ee} is the collision time for slow electrons of speed v interacting with Maxwellian ''field'' electrons of temperature T_{0f},[6]

$$t_E^{ee}=\frac{m_e^2 v^3}{16\pi e^4 n_e \ln\Lambda}\,\frac{3\sqrt{\pi}v_{tf}}{4v}\approx\frac{1}{4}\left(\frac{v_0}{v_{tf}}\right)^2\tau_{\rm col}. \tag{8}$$

Here t_E^{ee} has been rewritten in terms of $\tau_{\rm col}$ by using Eq. (3) with $\langle E\rangle\approx(3/2)T_{0f}=(3/4)m_e v_{tf}^2$.

Within a time period t_E^{ee}, the density of electrons which must be boosted in energy to prevent them from occupying the depleted region below $v=v_0$ will be comparable to the normal Maxwellian population of that region of velocity space,

$$n_{\rm slow}\sim\left(\frac{n_e}{\pi^{3/2}v_{tf}^3}\right)\left(\frac{4}{3}\,\pi v_0^3\right)=\frac{4}{3\sqrt{\pi}}\,n_e\left(\frac{v_0}{v_{tf}}\right)^3. \tag{9}$$

If the distribution were allowed to relax for a time t_E^{ee}, the number of slow electrons would approach this equilibrium value but would still be less than it, so $n_{\rm slow}$ will actually be somewhat less than the value on the right-hand side of Eq. (9).

Slow electrons must be boosted up high enough in the velocity distribution that they will not immediately return to the depleted region. The exact amount of energy which they must be given is not readily apparent in this simple model, but it should be comparable to the mean electron energy, $E_{\rm slow}\sim\langle E\rangle$.

Putting all of this information together, one arrives at the conclusion that

$$P_{\rm recirc}\sim\frac{v_0}{v_{tf}}\,\frac{n\langle E\rangle}{\tau_{\rm col}}. \tag{10}$$

The numerical coefficient by which this expression should be multiplied will be found from the rigorous derivation.

B. Rigorous derivation

Consider a fairly general isotropic particle velocity distribution $f(v)$ (for $v\geqslant 0$) which peaks at some speed v_0 and possesses characteristic widths v_{ts} and v_{tf} on the slow and fast sides of the peak, respectively:

$$f(v)\equiv\begin{cases} nK\{\exp[-(v-v_0)^2/v_{ts}^2]+\exp[-(v+v_0)^2/v_{ts}^2]\} & \text{for } v<v_0,\\ nK\{\exp[-(v-v_0)^2/v_{tf}^2]+\exp[-(v+v_0)^2/v_{ts}^2]\} & \text{for } v\geqslant v_0.\end{cases} \tag{11}$$

The normalization constant K is determined by the usual relation $\int f(v)4\pi v^2 dv=n$.

This distribution function, which is graphed in Fig. 1(a), has many virtues. It can be set to a Maxwellian by the choice $v_0=0$, and even for other values of v_0 it goes to the Maxwellian limit for large v. By varying the relative values of v_0, v_{ts}, and v_{tf}, a wide variety of distribution shapes may be studied. For example, for $v_t\equiv v_{ts}=v_{tf}$, Eq. (11) reduces to the beam-like distribution discussed in Sec. II A, as illustrated in Fig. 1(b). Alternatively, for $v_{ts}\ll v_0\ll v_{tf}$, Eq. (11) models the nearly Maxwellian electron distribution in which the slow electrons are depleted, as was also discussed in Sec. II A and is shown in Fig. 1(c). Yet despite this high degree of flexibility, the particular form of the distribution function in Eq. (11) allows one to obtain exact expressions for quantities such as the mean particle energy and the collision operator.

For an isotropic but otherwise general distribution function undergoing self-collisions, the collision operator may be written as[6,9]

$$\begin{aligned}\left(\frac{\partial f}{\partial t}\right)_{\rm col}&=-\nabla_{\mathbf v}\cdot\mathbf J\\ &=\frac{8\pi^2(Ze)^4\ln\Lambda}{m^2}\left\{\frac{2}{3}\,\frac{\partial^2 f}{\partial v^2}\left[\frac{1}{v^3}\int_0^v du\, f(u)u^4\right.\right.\\ &\quad\left.+\int_v^\infty du\, f(u)u\right]+2[f(v)]^2+\frac{4}{3v}\,\frac{\partial f}{\partial v}\\ &\quad\times\left[\int_0^\infty du\, f(u)u-\int_0^v du\, f(u)u\left(1-\frac{u}{v}\right)^2\right.\\ &\quad\left.\left.\times\left(1+\frac{u}{2v}\right)\right]\right\},\end{aligned} \tag{12}$$

in which $\mathbf J(\mathbf v)$ is the collisional velocity-space particle flux:[6]

$$\begin{aligned}\mathbf J(\mathbf v)=&-\frac{16\pi^2(Ze)^4\ln\Lambda}{m^2}\left\{\frac{\partial f}{\partial v}\,\frac{1}{3}\left[\frac{1}{v^3}\int_0^v du\, f(u)u^4\right.\right.\\ &\left.\left.+\int_v^\infty du\, f(u)u\right]+f\,\frac{1}{v^2}\int_0^v du f(u)u^2\right\}\hat{\mathbf v},\end{aligned} \tag{13}$$

where $\hat{\mathbf v}$ denotes the ''radial'' direction in velocity space.

Note that the inclusion of the second term on each line of Eq. (11) ensures that $[\partial f/\partial v]_{v=0}=0$ and $J(v=0)=0$, as is required for a self-consistent spherically symmetric distribution.

By using these expressions for $(\partial f/\partial t)_{\rm col}$ and $\mathbf J$, one may determine the minimum recirculating power density $P_{\rm recirc}$ needed to hold the non-Maxwellian distribution function of

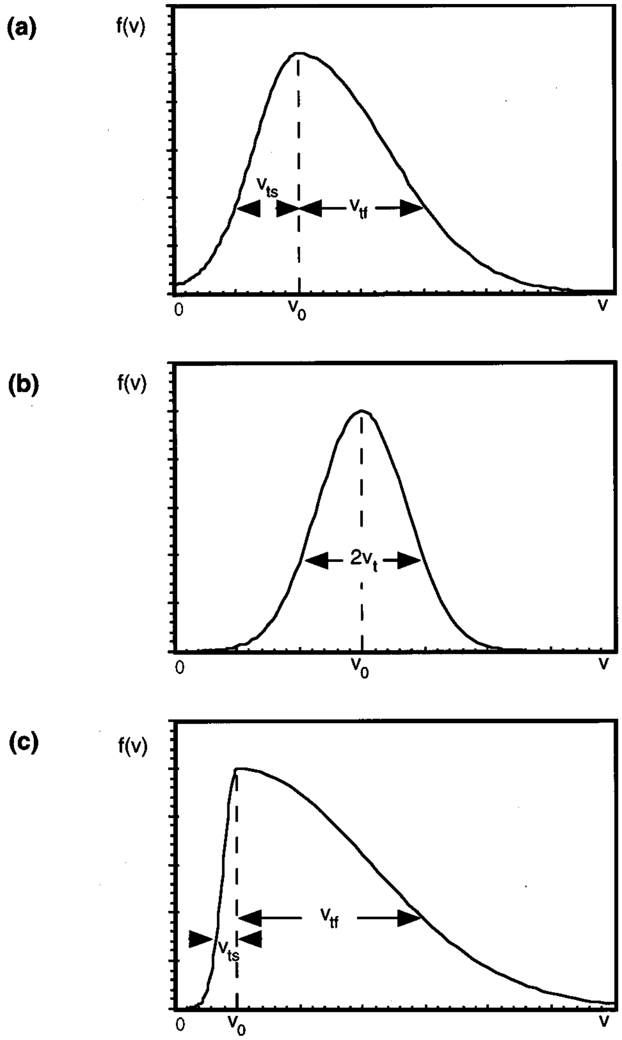

FIG. 1. Graphs of the isotropic velocity distribution of Eq. (11). The roles of the variables v_0, v_{ts}, and v_{tf} are illustrated in (a). For $v_t \equiv v_{ts} = v_{tf}$ as shown in (b), Eq. (11) describes an isotropic beam-like distribution. For $v_{ts} \ll v_0 \ll v_{tf}$ as illustrated in (c), Eq. (11) describes a nearly Maxwellian distribution in which the very slow particles have been depleted.

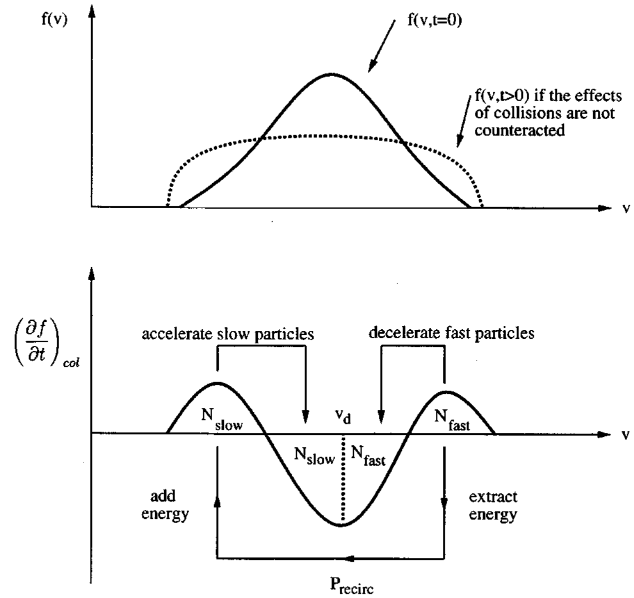

FIG. 2. A schematic diagram showing how to calculate the minimum recirculating power required to maintain a given non-Maxwellian isotropic velocity distribution shape. This particular example shows the recirculating power needed to sustain a distribution qualitatively similar to that in Fig. 1(b), but this general method may be extended to any isotropic but otherwise arbitrary velocity distribution, as described in Eq. (14).

Eq. (11) constant despite self-collisions. Figure 2 shows a simple case in which the collisional velocity-space flux is positive above and negative below some dividing velocity v_d. In other words, the dividing velocity is defined as the finite nonzero solution of the equation $J(v_d)=0$. If N_{slow} is the number of particles which become too slow in a given time period, the particles must be boosted up to the lowest N_{slow} number of vacant states in the desired distribution shape. Likewise, the N_{fast} particles which have become too fast must be decelerated to fill in the remaining vacant states on the other side of the dividing velocity. If energy losses from the distribution are neglected, the input power needed to accelerate the slow particles may theoretically be entirely obtained by extracting from the fast particles the exact amount of power needed to slow them down. This power is the minimum theoretical recirculating power.

For a general, isotropic distribution (not restricted to the distributions shown in Figs. 1 and 2), the appropriate mathematical definition for the minimum recirculating power density required to hold that distribution constant despite like-particle collisions is

$$P_{\mathrm{recirc}} \equiv \int_0^\infty (dv\,4\pi v^2)\left(\frac{1}{2}\,mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} \Theta[J(v)], \tag{14}$$

where Θ is the unit step function. The physical meaning of Eq. (14) is that the excess energy gained in collisions must be removed from particles upscattered in velocity-space regions where $J(v)>0$. This energy can then be given to collisionally down-scattered particles in regions where $J(v)<0$.

For the distribution like that in Fig. 2, Eq. (14) reduces to

$$\begin{aligned} P_{\mathrm{recirc}} &= \int_{v_d}^\infty (dv\,4\pi v^2)\left(\frac{1}{2}\,mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} \\ &= -\int_0^{v_d} (dv\,4\pi v^2)\left(\frac{1}{2}mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}}, \end{aligned} \tag{15}$$

in which v_d may be found from the equation

$$\int_0^{v_d} (dv\,4\pi v^2)\left(\frac{\partial f}{\partial t}\right)_{\mathrm{col}} = 0. \tag{16}$$

By Gauss's divergence theorem and the relation $(\partial f/\partial t)_{\mathrm{col}} = -\boldsymbol{\nabla}_{\mathbf{v}} \cdot \mathbf{J}$, Eq. (16) may be seen to be simply a restatement of the earlier condition on v_d, $J(v_d)=0$.

Equation (14) may be integrated numerically and expressed in terms of the density n, mean particle energy $\langle E \rangle$, and collision time τ_{col}.

For the important special case of Eq. (11) in which $v_{ts} \ll v_0 \ll v_{tf}$, it is found that

TABLE I. Selected values of the function $R_0(v_0/v_{tf})$.

v_0/v_{tf}	$R_0(v_0/v_{tf})$
1/60	0.0637
1/30	0.0644
1/10	0.0687
1/6	0.0749
1/3	0.0957
1	0.183

TABLE II. Selected values of the function $R_1(v_0/v_t)$.

v_0/v_t	$R_1(v_0/v_t)$
0.01	5.81×10^{-7}
0.1	5.63×10^{-4}
0.5	0.0365
1	0.0854
3	0.148
10	0.221
30	0.253
100	0.265

$$P_{\text{recirc}}=R_0(v_0/v_{tf})\left(\frac{v_0}{v_{tf}}\right)\frac{n\langle E\rangle}{\tau_{\text{col}}}, \tag{17}$$

where R_0 is a slowly varying function whose values are given in Table I.

Similarly, the recirculating power for the distribution of Eq. (11) in the beam-like case with $v_t\equiv v_{ts}=v_{tf}$ is

$$P_{\text{recirc}}=R_1(v_0/v_t)\left(\frac{v_0}{v_t}\right)\frac{n\langle E\rangle}{\tau_{\text{col}}}, \tag{18}$$

in which R_1 is a slowly varying function described in Table II.

The recirculating power may be compared with the fusion power. It will be assumed that there are two fuel ion species present, one of which is an isotope of hydrogen; x denotes the ratio of the density of the hydrogen isotope to the density of the second ion species, and Z_{i2} represents the charge state of the second ion species. The fusion power density may then be written as[1]

$$P_{\text{fus}}=1.602\times10^{-19}\,\frac{x}{(x+Z_{i2})^2}\,n_e^2\langle\sigma v\rangle_{\text{fus}}E_{\text{fus,eV}}\,\frac{\text{W}}{\text{cm}^3}, \tag{19}$$

where n_e is the electron density, $\langle\sigma v\rangle_{\text{fus}}$ is the average fusion reactivity in cm^3/s, and $E_{\text{fus,eV}}$ is the energy in eV released per reaction. If there is only one fuel ion species (which may or may not be a hydrogen isotope), the factor $x/(x+Z_{i2})^2$ in Eq. (19) should be replaced by $1/2Z_i^2$.[1]

For the case in which particle species "a" is kept non-Maxwellian with $v_t\equiv v_{ts}=v_{tf}$, the recirculating power compared with the fusion power is

$$\frac{P_{\text{recirc}}}{P_{\text{fus}}}=5.34\times10^{-6}R_1(v_0/v_t)\left(\frac{v_0}{v_t}\right)\sqrt{\frac{m_e}{m_a}}\times\frac{(x+Z_{i2})^2}{x}\,\frac{Z_a^4n_a^2}{n_e^2}\,\frac{\ln\Lambda}{\langle\sigma v\rangle_{\text{fus}}E_{\text{fus,eV}}\sqrt{\langle E_{a,\text{eV}}\rangle}}, \tag{20}$$

To apply Eq. (20) when $v_{ts}\ll v_0\ll v_{tf}$, one should make the substitutions $v_t\rightarrow v_{tf}$ and $R_1\rightarrow R_0$ in that equation.

Although these calculations of the minimum recirculating power apply to any possible means of recycling the power and are not restricted to a particular method, it may be helpful to give specific examples of systems for recycling the power. In principle, power may be selectively extracted from particular velocity-space regions of the particle distributions via high-voltage charged particle direct electric converters, electromagnetic radiation from the particles, or other means. This power may then (in principle) be processed and reinjected into other velocity-space regions of the particle distributions by employing cyclotron resonance heating, particle beam injection, electromagnetic acceleration, or other methods.

Due to the difficulties of precisely manipulating particles in narrowly defined regions of velocity space, realistic systems for recirculating the power will probably have to recycle considerably more than the minimum theoretical recirculating power. Furthermore, realistic systems will involve unlike particle collisions and instabilities that tend to increase the minimum required recirculating power, and all specific foreseeable systems will also lose a significant amount of the power in the process of recirculating it. Of course, real fusion reactors will have many other power loss mechanisms as well. Aside from the actual losses on the recirculating power, simply having to recycle an amount of power comparable to or greater than the fusion power would make the reactor technologically cumbersome and relatively unattractive as a commercial power source. Because of all of these reasons, a nonequilibrium reactor design which is to be considered promising should probably have a minimum recirculating power that is at least one order of magnitude smaller than the gross fusion power.

C. Results

In judging the performance of fusion systems, bremsstrahlung radiation will be considered. The bremsstrahlung power loss density, including relativistic corrections, is given in[1]

$$P_{\text{brem}}=1.69\times10^{-32}n_e^2\sqrt{T_e}\left\{\frac{\Sigma_iZ_i^2n_i}{n_e}\left[1+0.7936\,\frac{T_e}{m_ec^2}+1.874\left(\frac{T_e}{m_ec^2}\right)^2\right]+\frac{3}{\sqrt{2}}\,\frac{T_e}{m_ec^2}\right\}\frac{\text{W}}{\text{cm}^3}, \tag{21}$$

in which the electron temperature T_e and rest energy m_ec^2 are in eV.

The D–T, D–^{3}He, and D–D fuels can theoretically produce net power when they are burned in a plasma which is essentially in thermodynamic equilibrium. Such systems will not be considered here, since their optimum performance is discussed in detail in Refs. 1 and 5. Unfortunately, although the minimum bremsstrahlung power loss from such systems is in principle tolerably small in comparison with the fusion power, for D–^{3}He and D–D it is not as small as one might wish. Furthermore, for ^{3}He–^{3}He, p-^{11}B, and p-^{6}Li plasmas

TABLE III. Comparison of recirculating power and bremsstrahlung radiation power with gross fusion power for nearly Maxwellian electron distributions with the slow electrons depleted. (The ions are Maxwellian.)

Fuel mixture	$\langle E_i \rangle$ (keV)	$\langle E_e \rangle$ (keV)	$\langle \sigma v \rangle_{\rm fus}$ (10^{-16} cm^3/s)	$E_{\rm fus}$ (MeV)	$P_{\rm brem}/P_{\rm fus}$	$P_{\rm recirc}/P_{\rm fus}$
D–^{3}He (1:1)	150	39	1.67	18.3	0.093	5.2
D–D	750	170	1.90	3.7	0.18	2.6
^{3}He–^{3}He	1500	160	1.25	12.9	0.50	5.6
p-^{11}B (5:1)	450	35	2.39	8.7	0.50	52
p-^{6}Li(3:1)	1200	22	1.60	4.0	0.50	330

TABLE IV. Comparison of recirculating power and bremsstrahlung radiation power with gross fusion power for isotropic, beam-like electron distributions. (The ions are Maxwellian. For DT, $\langle \sigma v \rangle_{\rm fus}=8.54\times10^{-16}$ cm^3/s and $E_{\rm fus}=17.6$ MeV.)

Fuel mixture	$\langle \overline{E_i} \rangle$ (keV)	$\langle E_e \rangle$ (keV)	$P_{\rm brem}/P_{\rm fus}$	$P_{\rm recirc}/P_{\rm fus}$ ($v_0/v_t=1$)	$P_{\rm recirc}/P_{\rm fus}$ ($v_0/v_t=10$)
D–T(1:1)	75	63	0.007	7.3	190
D–^{3}He (1:1)	150	108	0.19	61	1600
D–D	750	315	0.35	35	900
^{3}He–^{3}He	1500	160	0.50	85	2200
p-^{11}B (5:1)	450	35	0.50	350	9100
p-^{6}Li (3:1)	1200	22	0.50	870	23 000

which are essentially in thermodynamic equilibrium, the bremsstrahlung radiation losses are prohibitively large.[1]

Since ion–electron energy transfer is mediated by the comparatively small number of electrons which are moving more slowly than the ions,[6] one obvious method of lowering the electron temperature and hence the radiation losses in an advanced fuel reactor would be to actively deplete those slow electrons. An appropriate non-Maxwellian electron distribution may be described by Eq. (11) with $v_{ts} \ll v_0 \ll v_{tf}$, where $v_0 \approx 2v_{ti1} \equiv 2\sqrt{2T_i/m_{i1}}$ (in which the $i1$ species is the lighter of the two fuel ion species) and $v_{tf} \sim v_{te} \equiv \sqrt{2T_e/m_e}$.

Table III summarizes the minimum recirculating power requirements needed to maintain such an electron distribution shape and lower the bremsstrahlung radiation losses $P_{\rm brem}$ for several different fusion fuels. For this and subsequent tables, the ion energies and fuel mixtures have been chosen to approximately minimize the ratio of bremsstrahlung losses to fusion power, and the Coulomb logarithm has been set at 15, an optimistic value for a magnetic fusion reactor. (The lower Coulomb log of inertial confinement fusion does not alter the results enough to change this paper's conclusions about the viability of various fusion approaches.)

For D–^{3}He and D–D, the electron energies in Table III have been chosen to reduce the bremsstrahlung losses to half of what they would be in the equilibrium state,[1] and for the other fuels the electron energies have been chosen to limit the bremsstrahlung to half of the fusion power. Fusion reactivities are drawn from Ref. 10. D–T is not included in the table, since its radiation losses can in theory be made quite small even with perfectly Maxwellian electrons. As shown in the table, the recirculating power levels are substantially larger than the fusion power. If the mean electron energy is lowered below the values in the table, the recirculating power will increase; if the electron energy is raised, the bremsstrahlung losses will increase. More precise tailoring of the electron distribution shape can lower the recirculating power levels somewhat,[5] but the improvement is far from being large enough to be truly useful. Therefore, all of the systems in Table III fail to meet the criterion for a promising nonequilibrium reactor concept as defined above.

(As discussed in the Introduction, the ultimate goal of this investigation is to examine the cleanest possible fusion approaches. Therefore, in these calculations it has been assumed that the fusion products are somehow removed from the plasma before they can undergo any further reactions, in order to prevent additional neutron production and radioactivity from reactions of daughter nuclei. Leaving the fusion products in the plasma would substantially alter the performance of only two of the fuels. The effective $E_{\rm fus}$ and $P_{\rm fus}$ for D–D would increase by a factor of 5.85 due to burnup of bred T and ^{3}He, but then large numbers of unpleasant 14 MeV D–T neutrons would be produced. Also, this performance increase would not be large enough to make most of the D–D systems considered in this paper truly feasible. Allowing the ^{3}He bred by p-^{6}Li to burn up with exogenous D would effectively improve the performance of p-^{6}Li by a factor of 5.5, but it would increase the neutron production while still not rendering the p-^{6}Li systems considered in this paper feasible.)

Similarly, Tables IV and V reveal the difficulty of maintaining nearly monoenergetic velocity distributions ($v_t \equiv v_{ts} = v_{tf}$ and $v_0 \geqslant v_t$) for electrons and ions, respectively. Such beam-like distributions have been proposed for use in a number of different nonequilibrium fusion approaches, such as inertial-electrostatic confinement,[11] migma,[12] and related ideas.[13–15] The recirculating power levels for beam-like electrons are clearly prohibitive for all of the cases in Table IV, regardless of the degree of sharpness of the distribution peaks. Of all of the beam-like ion cases considered in Table V, only D–T plasmas might be able to operate with an acceptable recirculating power level (see Ref. 15 for an example), and even then only when the total ion population does not deviate too greatly from thermodynamic equilibrium. [For D–T and D–^{3}He in Table V, Eq. (20) has been used to estimate the effects of collisions between unlike ions as well as those between like ions by taking into account the differences in mass and charge between the species.]

TABLE V. Comparison of recirculating power with gross fusion power for isotropic, beam-like ion distributions. (The electrons are Maxwellian.)

Fuel mixture	$\langle E_i \rangle$ (keV)	$P_{\rm recirc}/P_{\rm fus}$ ($v_0/v_t=2$)	$P_{\rm recirc}/P_{\rm fus}$ ($v_0/v_t=10$)
D–T (1:1)	75	0.3	3
D–^{3}He (1:1)	150	2	20
D–D	750	1.1	9.6
^{3}He–^{3}He	1500	4.3	38

TABLE VI. Comparison of recirculating power and bremsstrahlung radiation power with gross fusion power for the active refrigeration of electrons. (The ions and electrons are Maxwellian.)

Fuel	$\langle E_i \rangle$ (keV)	$\langle E_e \rangle$ (keV)	$P_{\text{brem}}/P_{\text{fus}}$	$P_{\text{recirc}}/P_{\text{fus}}$
D–^{3}He (1:1)	150	39	0.093	1.9
D–D	750	170	0.18	0.9
^{3}He–^{3}He	1500	160	0.50	6.2
p-^{11}B (5:1)	450	35	0.50	33
p-^{6}Li (3:1)	1200	22	0.50	320

III. DIFFERENT PARTICLE SPECIES AT RADICALLY DIFFERENT MEAN ENERGIES

As has been mentioned, it would be desirable to keep the mean electron energy much lower than the mean ion energy in an advanced fuel reactor, in order to minimize the bremsstrahlung and synchrotron losses. If the ion temperature is held constant and Coulomb friction with the ions is the only energy source available to the electrons, the electron temperature will equilibrate to a somewhat lower value than the ion temperature, since the electrons lose energy by radiation.[1] A hypothetical system for keeping the electron temperature lower than this equilibrium value would have to continually extract a minimum recirculating power from the electrons and return it to the ions in order to keep the ions and electrons ''decoupled'' in energy. In this case the minimum recirculating power is $P_{\text{recirc}} \equiv P_{ie} - P_{\text{brem}}$, where the ion–electron energy transfer rate P_{ie} is given in Ref. 1,

$$P_{ie} = 7.61 \times 10^{-28} n_e \left(1 + \frac{0.3 T_e}{m_e c^2} \right) \sum_i \frac{Z_i^2 n_i m_p}{m_i T_e^{3/2}} \ln \Lambda \times \exp\left[- \left(3.5 \sum_i \frac{Z_i^2 n_i}{n_e} \frac{m_e}{m_i} \frac{T_i}{T_e} \right)^{2/3} \right] (T_i - T_e) \ \frac{\text{W}}{\text{cm}^3}, \quad (22)$$

in which m_p is the proton mass, the temperatures and electron rest energy are in eV, and the Coulomb logarithm is $\ln \Lambda \approx 24 - \ln(\sqrt{n_e}/T_e)$.

Table VI gives the recirculating power levels required to lower the electron temperature in various fuel mixtures enough that the bremsstrahlung radiation losses will be substantially reduced from their usual equilibrium values. For each of the fuels listed in the table, the amount of power which must be recycled is clearly much too large in comparison with the fusion power. Methods of passively[6] or actively (see the previous section) depleting the slow electrons to reduce the ion–electron energy transfer rate and hence the required recirculating power are insufficient to improve the outlook for ion–electron energy decoupling. Likewise, all other presently available techniques are unable to reduce the recirculating power to manageable levels.[5] Thus fusion systems that attempt to actively cool the electrons can be ruled out.

A related idea would be to maintain two fuel ion species at significantly different temperatures in order to boost the fusion reaction rate or suppress undesirable side reactions. Unfortunately, the temperatures of two ion species equilibrate on the order of $\sqrt{m_i/m_e}$ faster than the temperatures of ions and electrons interacting with each other,[16] so attempts at energy decoupling between two ion species meet with the same fate as ion–electron energy decoupling, as shown explicitly in Refs. 1 and 5. All currently available techniques for potentially decreasing the energy transfer rate between the ion species and lowering the recirculating power levels are insufficient for the present task.[5] Therefore, fusion systems which attempt to decouple the relative energies of two fuel ion species do not appear to be feasible.

It has not actually been necessary to assume that the plasma is in steady state, either for these cases of interspecies energy differences or for the earlier situations with non-Maxwellian distributions. For virtually all of the cases considered, it has been shown that the power flow in the plasma's phase space corresponding to particle species adjusting their relative energies and velocity distribution shapes is considerably larger than the fusion power. As a result, even pulsed systems in which the plasma is actively reordered back to the desired nonequilibrium state at the end of each pulse would not be useful; the power involved in reordering the plasma for the next pulse would exceed the fusion power derived from the pulse.

IV. IMPLICATIONS FOR ADVANCED-FUEL FUSION

Because of the neutron production and radioactive inventory associated with D–T and D–D fusion, it has been observed that fusion reactors could be made much more desirable if cleaner, more advanced fusion fuels could be used.[2]

If they could be successfully employed, the advanced aneutronic fuels (^{3}He–^{3}He, p-^{11}B, and p-^{6}Li) would be very attractive reactor fuels due to the very low neutron production and radioactive inventories associated with them. Unfortunately, there appears to be no way to produce net power with any of these fuels. If they are burned in a plasma which is essentially in thermodynamic equilibrium, the electron temperature and hence the radiation losses will be too large.[1] As revealed in Table VI, actively cooling the electrons while maintaining the ion temperature by somehow recirculating power from the electrons back to the ions would require one to recycle much more power than the fusion power, regardless of the specific mechanism for actually returning the power. An alternate method of lowering the electron temperature would be to actively deplete the very slow electrons that mediate ion–electron energy transfer, but this technique would still require prohibitively large amounts of recirculating power, as shown in Table III. As has been discussed, the power recycling requirements also rule out boosting the fusion reaction rate over the Maxwellian-averaged value by keeping one fuel ion species at a substantially different mean energy than the other.

D–^{3}He is a fusion fuel which can break even against radiation losses in an equilibrium plasma, but it is plagued by D–D side reactions which produce neutrons and tritium and thus keep the fuel from being as clean as would be desired. In an equilibrium plasma, operating much more ^{3}He-rich than a 1:1 fuel mixture in order to suppress D–D reactions leads to radiation losses which are too large and a fusion power which is too small. Lowering the electron temperature and bremsstrahlung losses by active cooling of the electrons (Table VI) or by active depletion of the very slow electrons (Table III) in order to permit the use of more ^{3}He-rich fuel mixtures would require intolerably large amounts of recirculating power. Likewise, using highly nonequilibrium ion populations in order to improve the ratio of the D–^{3}He and D–D reactivities would also involve the recirculation of too much power in comparison with the fusion power, as has been discussed.

Therefore, the large amounts of power which must be recycled in order to sustain nonequilibrium fusion plasmas prevent such systems from being useful for burning the relatively clean advanced aneutronic fuels or for reducing the radioactivity of D–^{3}He fusion.

Furthermore, the results of this paper indicate that fusion approaches such as inertial-electrostatic confinement,[11] migma,[12] and other ideas[13,14] which attempt to employ highly nonequilibrium plasmas will probably not even be able to produce net power with D–T, and they certainly will not be able to produce net power with any of the other fusion fuels. (Most of these proposed approaches do not even have mechanisms for recirculating power to stay out of equilibrium, and so they would quickly relax to equilibrium.[1] Even if they had such mechanisms, those mechanisms would be limited by the constraints found in this paper.) This fundamental and broadly applicable limitation is in addition to certain design-specific flaws which have already been noted with some of these approaches.[1,17]

Some observations should also be made regarding the connections between this paper's results and the method proposed by Snyder *et al.*[18] for channeling fusion product energy to fuel ions. Most of the performance improvement reported in Ref. 18 comes from reducing the heating of electrons by mechanisms other than Coulomb friction with the fuel ions. In comparison, this paper has assumed from the outset that there is *no* heating of electrons by mechanisms other than friction with the fuel ions, and the paper has proceeded to examine broad categories of approaches for improving reactor performance still further. Thus this paper has focused on potential approaches beyond those which are proposed in Ref. 18; the results of this paper, as discouraging as they may seem, are actually inherently more optimistic than the results reported by Snyder *et al.*

The numerical results in Ref. 18 also show that in the absence of an active particle cooling system, two fuel ion species cannot be kept at substantially different mean energies, as shown analytically in Ref. 1. Furthermore, the numerical results of Snyder *et al.* demonstrate that changing from Maxwellian to non-Maxwellian ion distributions would alter the fusion reactivity by at most a few percent, provided that the ions are at the same mean energy; this change is in agreement with the results in Ref. 5 and is much too small to alter this paper's conclusions about the viability of various fusion approaches.

V. CONCLUSIONS

In this paper we have derived fundamental power limitations that apply to virtually any possible type of fusion reactor in which the electrons or fuel ions possess a significantly non-Maxwellian velocity distribution or in which two major particle species are at radically different mean energies. Analytical Fokker–Planck calculations have been used to accurately determine the minimum recirculating power that must be extracted from undesirable regions of the plasma's phase space and reinjected into the proper regions of the phase space in order to counteract the effects of collisional scattering events and keep the plasma out of thermodynamic equilibrium. In virtually all cases, this minimum recirculating power is substantially larger than the fusion power, so barring the discovery of methods for recirculating the power at exceedingly high efficiencies, reactors employing substantially nonequilibrium plasmas will not be able to produce net power.

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[1] T. H. Rider, Phys. Plasmas **2**, 1853 (1995).
[2] L. M. Lidsky, J. Fusion Energy **2**, 269 (1982).
[3] E. A. Frieman, M. L. Goldberger, K. M. Watson, S. Weinberg, and M. N. Rosenbluth, Phys. Fluids **5**, 196 (1962).
[4] H. P. Furth, Phys. Fluids **6**, 48 (1963).
[5] T. H. Rider, Ph.D. thesis, Massachusetts Institute of Technology, 1995.
[6] T. H. Rider and P. J. Catto, Phys. Plasmas **2**, 1873 (1995).
[7] D. V. Sivukhin, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4, pp. 93–241.
[8] D. L. Book, *NRL Plasma Formulary* (Naval Research Laboratory, Washington, DC, revised 1987).
[9] W. M. MacDonald, M. N. Rosenbluth, and W. Chuck, Phys. Rev. **107**, 350 (1957).
[10] See National Technical Information Service Document No. ORNL TM-6914 (J. R. McNally, Jr., K. E. Rothe, and R. D. Sharp, *Fusion Reactivity Graphs and Tables for Charged Particle Reactions*, Oak Ridge National Laboratory Report ORNL TM-6914, 1979). Copies are available from the National Technical Information Service, Springfield, VA 22161.
[11] R. W. Bussard and N. A. Krall, Fusion Tech. **26**, 1326 (1994).
[12] B. C. Maglich, Nucl. Instrum. Methods A **271**, 13 (1988).
[13] D. C. Barnes, R. A. Nebel, and L. Turner, Phys. Fluids B **5**, 3651 (1993).
[14] N. Rostoker, F. Wessel, H. Rahman, B. C. Maglich, B. Spivey, and A. Fisher, Phys. Rev. Lett. **70**, 1818 (1993).
[15] J. M. Dawson, H. P. Furth, and F. H. Tenney, Phys. Rev. Lett. **26**, 1156 (1971).
[16] L. Spitzer, Jr., *Physics of Fully Ionized Gases*, 2nd revised ed. (Wiley–Interscience, New York, 1962).
[17] W. M. Nevins, Phys. Plasmas **2**, 3804 (1995).
[18] P. B. Snyder, M. C. Herrmann, and N. J. Fisch, J. Fusion Energy **13**, 281 (1994).